CVBEM MODELING OF TRACKING TWO-DIMENSIONAL FREEZING FRONTS IN ALGID SOIL

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ABSTRACT

The use of the Complex Variable Boundary Element Method (CVBEM) is a new numerical approach for two-dimensional Laplace and Poisson Problems. The CVBEM can be formulated either by the Cauchy integral theorem or by the generalized Fourier series analysis. A major benefit in the use of the CVBEM over other numerical methods is the accurate and easy-to-use "approximate boundary" error evaluation technique. The CVBEM apprach can be used directly in engineering applications, or used to provide a wide range of highly accurate approximations for two-dimensional phase change problems (where the freezing front movement is slow) for checking modeling results produced by other numerical methods.

INTRODUCTION

The use of the Complex Variable Boundary Element Method (CVBEM) to model soil-water phase change effects is a new numerical approach to this class of problems. In previous work, Hromadka et al.(7) compared the CVBEM solution to a domain solution method and prototype data for the Deadhorse Airport runway at Prudhoe Bay, Alaska, and Hromadka and Berg(4) applied the CVBEM to the problem of predicting freezing fronts in two-dimensional soil systems. In another work, the model is further extended to include an approximation of soil-water flow (Hromadka and Guymon(5)). In contrast to the CVBEM approach, an example in the use of real variable boundary element methods (Brebbia(1)) in the approximation of such moving boundary phase change problems and a review of the pertinent literature is given in O'Niell (8).

Hromadka and Guymon(6) developed an error estimation scheme which exactly evaluates the error distribution on the problem boundary with results from the CVBEM approximator matching the known boundary conditions. This error determination is used to add boundary nodes to improve accuracy. Thus, the CVBEM permits a direct and immediate determination of the approximation error involved in the solution of an assumed Laplacian system. The modeling accuracy is valuated by the determination of an approximate boundary upon which the CVBEM provides an exact solution. Although inhomogeneity (and anisotropy) can be included in the CVBEM model, the resulting fully-populated matrix system quickly becomes large. Therefore in this work, the domain is assumed homogeneous and isotropic except for differences in frozen and thawed conduction parameters for frozen and thawed regions, respectively.

Because the numerical technique is a boundary integral approach, the control volume thermal regime is modeled with respect to the boundary values and, therefore, the data entry requirements are significantly less than that usually required of domain methods such as finite-differences or finite-elements. Soil-water phase change along the freezing front is modeled as a simple balance between computed heat flux and the evolution of soil-water volumetric latent heat of fusion.

HEAT FLOW MODEL

For a wide range of soil freezing (or thawing) problems, the freezing front movement is sufficiently slow such that the governing heat flow equation can be modeled using a timestepped steady state heat flow approximation. That is for small durations of time, the heat flux along the freezing front can be computed assuming the tempera-distribution within the frozen (or thawed) regions are potential functions (i.e., the Laplace equation applies). Figure 1 illustrates a typical two-phase problem definition where the heat flow model solves for heat flux along the freezing front by solving the Laplace equation (by use of potential functions) in both the frozen and thawed regions.

To develop mathematical models of the Laplace equation in each region, a CVBEM approximator is generated which matches specified boundary conditions of either temperature or flux at nodal point locations on the problem boundary and freezing front. The CVBEM approximator exactly satisfies the Laplace equation; consequently there is no modeling error in solving the governing Laplace equation (heat flow model), there is only error in matching the boundary conditions continuously. Figure 2 shows an example roadway problem where the freezing front is initially located some known distance below the surface. Boundary conditions for the example

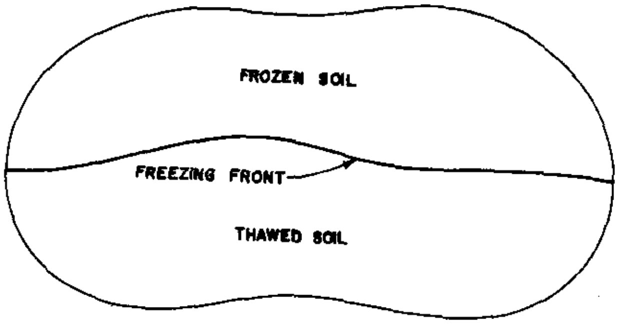

FIG. 1. TYPICAL TWO-PHASE PROBLEM DEFINITION

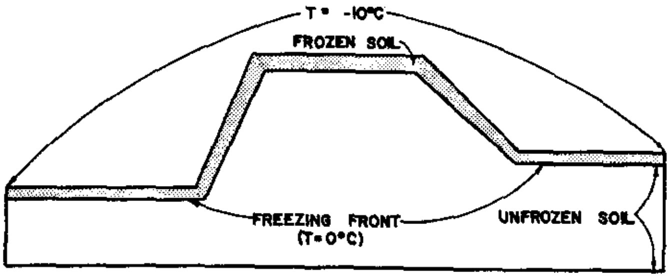

FIG. 2. TYPICAL ROADWAY EMBANKMENT PROBLEM

problem and a nodal point placement scheme are shown in Figure 3. The usual modeling procedure is to use the approximate boundary technique to analyze the initial conditions for model accuracy. After the analyst is satisfied with the CVBEM approximator and its associated level of accuracy then the program is executed to model the freezing front evolution.

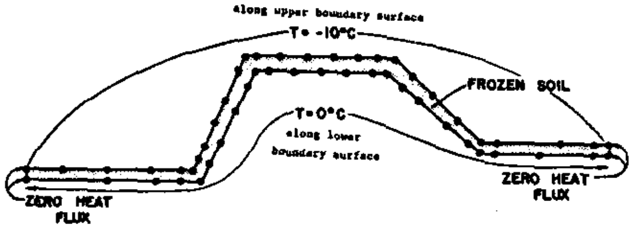

FIG. 3. NODAL POINT PLACEMENT AND BOUNDARY CONDITIONS FOR FIG. 2 PROBLEM

PHASE CHANGE MODEL

For each timestep, a CVBEM approximator is generated on the problem geometry and boundary conditions. Heat flux is computed along the freezing front using the CVBEM approximation stream function values. The heat flux estimates are assumed to directly relate to the freezing front. Consequently, a freezing process for the example of Fig. 3 results in a downward migration of the freezing front such that the product of the timestep and net heat flux equals the latent heat evolved by the change in freezing front coordinates.

Two freezing front displacements models are available:

(a) All displacement occurs in the vertical direction. This simplified model is generally appropriate for many roadway problems.

(b) All computed displacement are based on their outward normal vectors. This model is the most accurate, but requires additional computational effort than the vertical displacement model.

Figure 4 shows the nodal displacement in the outward normal direction.

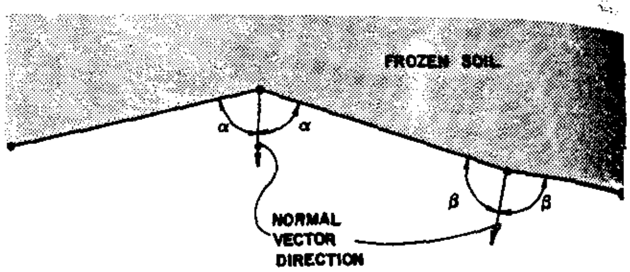

FIG. 4. NORMAL VECTOR COORDINATE DISPLACEMENT MODE

APPLICATION

Example 1: Nodal Density and Timestep Size Sensitivity Analysis

A sensitivity analysis is prepared examining different time increments and nodal point densities and the resulting effects on CVBEM modeling results. Figure 5 shows the different nodal densities and Table 1 shows the results from the several CVBEM Models. From the analysis, it appears that a small timestep (6-hours) is preferred, but a large timestep such as 60 hours results in an error with respect to the one-dimensional Stefan solution (Carslaw and Jaeger(2)) of only 2-percent. Additionally, a relatively sparse nodal density of only 30 nodes results in a satisfactory condition.

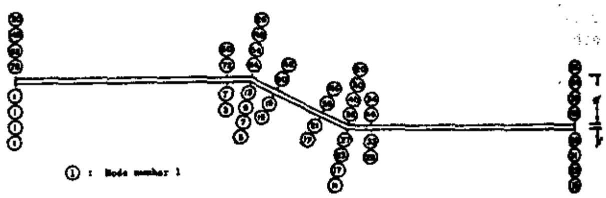

FIG. 5. NODAL POINT NUMBER FOR 4 CVBEM MODEL DENSITIES

Example 2: Comparison to Two-Dimensional Domain Modeling Results

The CVBEM modeling results for a roadway embankment problem are compared to results from a Nodal Domain Integration (NDI) two-dimensional phase change model (Hromadka et al.(7)) in Fig. 6. The NDI model is based upon an isothermal soil-water phase change approximation, and uses an apparent heat capacity approach to model the freezing front evolution in the fixed grid domain model. Figure 7 shows compatible results for both models.

Example 3: Chilled Pipeline Underneath a Roadway Embankment

Figure 8 depicts three submodels that were used in the CVBEM model to simulate the freezing/thawing front in the roadway embankment with a buried chilled pipeline.

Figure 9 shows the freezing/thawing front after 5 days of simulation. The approximated freezing front beneath the ground surface is close to the one-dimensional Stefan solution. The approximated freezing front around the chilled pipeline can be improved by adding more nodal points along the initial circumference of the freezing front.

Section / Timestep	A-A	B-B	C-C	D-D	Number of Nodes
6 hrs	1.3466 ft. (1.3459)	1.4645 ft. (1.4661)	1.2594 ft. (1.2632)	1.3466 ft. (1.3459)	78
12	1.3489 (1.3482)	1.4683 (1.4698)	1.2604 (1.2641)	1.3489 (1.3482)	78
24	1.3537 (1.3530)	1.4764 (1.4770)	1.2625 (1.2660)	1.3537 (1.3529)	78
60	1.3697 (1.3689)	1.5023 (1.4829)	1.2687 (1.2709)	1.3698 (1.3689)	78
6	*1.3466* (1.3459)	*1.4645* (1.4661)	*1.2594* (1.2632)	*1.3466* (1.3459)	78
6	1.3466 (1.3459)	1.4645 (1.4661)	1.2594 (1.2632)	1.3466 (1.3459)	62
60	1.3698 (1.3689)	1.5023 (1.4829)	1.2687 (1.2709)	1.3698 (1.3689)	62
6	1.3467 (1.3459)	1.4649 (1.4667)	1.2591 (1.2630)	1.3467 (1.3459)	46
60	1.3696 (1.3688)	1.5026 (1.4836)	1.2685 (1.2708)	1.3698 (1.3690)	46
6	1.3468 (1.3461)	1.4797 (1.4778)	1.2365 (1.2444)	1.3468 (1.3460)	30
60	1.3698 (1.3688)	1.5241 (1.4887)	1.2392 (1.2472)	1.3699 (1.3690)	30

Note:
Sections A-A, B-B, C-C, D-D are depicted on figure 7a.
1.3466: *Results from Vertical Displacement Model*
(1.3459): Results from Normal Vector Displacement Model
For example, Stefan solution is 1.344 ft.

TABLE 1 COMPARISON OF CVBEM MODEL RESULTS IN PREDICTING FREEZING FRONT LOCATION

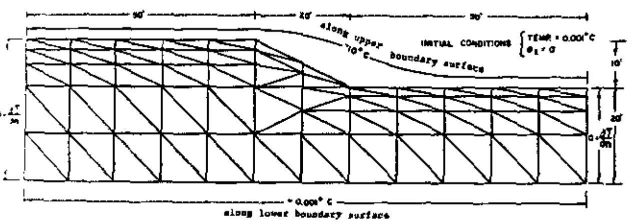

FIG. 6 EXAMPLE PROBLEM ROADWAY EMBANKMENT DISCRETIZED INTO FINITE ELEMENTS

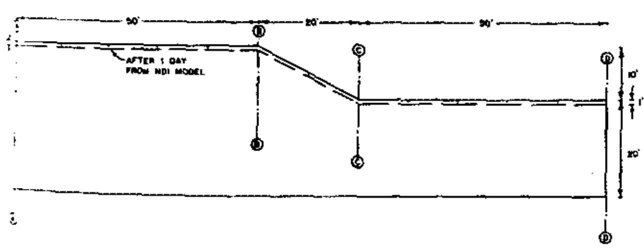

FIG. 7a INITIAL CONDITIONS AND CROSS SECTION LOCATIONS

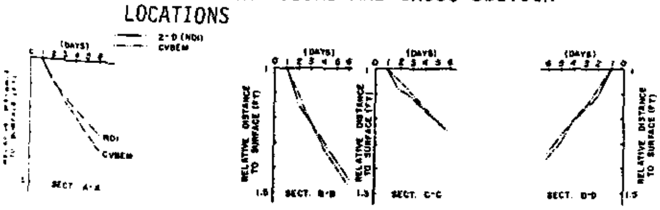

FIG. 7b COMPARISON OF CVBEM AND NDI MODELING RESULTS

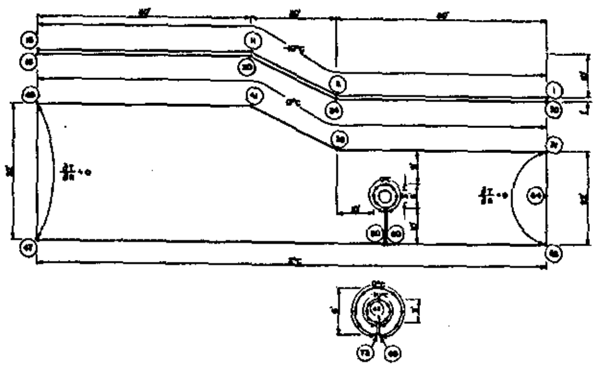

FIG. 8 INITIAL AND BOUNDARY CONDITIIONS FOR CHILLED PIPELINE PROBLEM

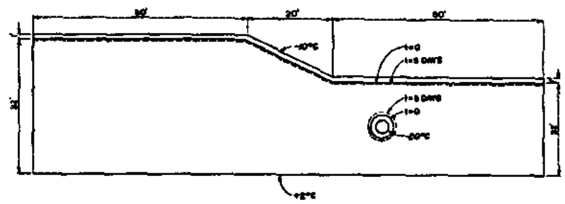

FIG. 9 FREEZING FRONT POSITION AFTER 5-DAYS OF SIMULATION FOR EXAMPLE 3

CONCLUSION

A major benefit in the use of the CVBEM over other numerical methods (including real variable boundary element methods and domain methods such as finite-differences and finite-elements) is the accurate and easy-to-use "approximate boundary" error evaluation technique. Other numerical methods can be evaluated for modeling error (where exact mathematical solutions do *not* exist) by increasing nodal point densities and comparing the resulting changes in predicted nodal values *of the governing equation's state variable. In contrast,* the CVBEM approximate boundary error evaluation technique is simply the process of locating the (x,y) points where the CVBEM approximate function meets the specified boundary condition values. Often, the CVBEM approximation analysis is terminated when the approximate boundary differs from the true problem boundary to within the construction tolerance of the project, resulting in an exact CVBEM model of a probable constructed *version of the engineered plan drawings. Consequently* the CVBEM approach can be used directly in engineering applications, or used to provide a wide range of highly accurate approximations for two-dimensional phase change problems (where the freezing front movement is slow) for checking modeling results produced by other numerical methods.

Because the numerical technique is a boundary integral approach, the control volume thermal regime is *modeled with respect to the boundary values, and there*fore, the data entry requirements are significantly less than that usually required of domain methods such as finite-differences or finite-elements.

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