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Two-Dimensional Model of Coupled Heat and Moisture Transport in Frost-Heaving Soils

A two-dimensional model of coupled heat and moisture flow in frost-heaving soils is developed based upon well known equations of heat and moisture flow in soils. Numerical solution is by the nodal domain integration method which includes the integrated finite difference and the Galerkin finite element methods. Solution of the phase change process is approximated by an isothermal approach and phenomenological equations are assumed for processes occurring in freezing or thawing zones. The model has been verified against experimental one-dimensional freezing soil column data and experimental two-dimensional soil thawing tank data as well as two-dimensional soil seepage data. The model has been applied to several simple but useful field problems such as roadway embankment freezing and frost heaving.

Introduction

Numerical modeling of coupled heat and moisture transport in freezing and thawing soils has been the subject of a number of investigations generally beginning in the mid-1970's (Harlan, [14] and Guymon and Luthin, [13]). Modeling efforts reported in the literature primarily deal with one-dimensional heat and moisture transport. These models and numerical models of frost heave are reviewed by Guymon, et al. [12], Hopke [15], and O'Niell [26] among others. While one-dimensional models of soil freezing or thawing are adequate for a large number of applications, two or three-dimensional models are required for many problems, e.g., buried pipelines, roadway berms, and embankments on permafrost.

The purpose of this paper is to present a dynamic two-dimensional numerical model of coupled heat and moisture transport which is generally applicable to freezing and thawing soils. The model includes the ability to estimate frost heave and thaw consolidation for simple but useful geometries. Some of the problems associated with such modeling exercises are reviewed.

Theory

The theory of heat transport in freezing soils and their thermal properties have been the subject of investigation since before the turn of the century, e.g., Stefan [27]. Recent definitive works on this topic are Lunardini [23] and Farouki [6]. As a result, the deterministic equations of heat transport in moist soils are well established; i.e., in two dimensions

$$\frac{\partial}{\partial x}\left[K_T\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_T\frac{\partial T}{\partial y}\right]=C_m\frac{\partial T}{\partial t}-L\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t}+C_w v_x\frac{\partial T}{\partial x}+C_w v_y\frac{\partial T}{\partial y} \quad (1)$$

where variables are described in the Nomenclature. The density parameters are relatively precisely known for modeling purposes. The latent heat parameter, L, is a function of temperature and salinity and can be assumed to be constant for dilute solutions with temperatures less than $-20°C$ (Anderson, et al. [1]). The latent heat term only contributes to equation (1) when soil regions are undergoing freezing or thawing. The remaining parameters, K and C_m, are functions of the volumetric fractions of each material constituent and soil structure, among other factors. There is some variation in computed parameters depending upon the method of measurement or computation. DeVries [5] method of estimating these parameters is often used, e.g., for C_m

$$C_m=\Sigma C_j\theta_j \quad (2)$$

where C_j = volumetric heat capacity of jth constituent and θ_j = volumetric fraction of jth constituent. A similar equation is used with and without a particle contact function for thermal conductivity. Equation (1) is nonlinear since the K_T and C_m parameters depend upon the amount of ice and water coexistent in a freezing and thawing soil. Additionally, equation (1) is coupled to equations describing the moisture state of a soil and soil freezing characteristic relationships such as discussed by Anderson, et al. [1].

Similarly the theory of water movement in unfrozen isothermal soils is well advanced; e.g., Bear [2]. A deterministic equation for moisture transport in nondeformable saturated or unsaturated porous media is

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$$\frac{\partial}{\partial x}\left[K_H \frac{\partial \phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_H \frac{\partial \phi}{\partial y}\right]=\frac{\partial \theta_u}{\partial t}+S \quad (3)$$

In unsaturated soils K_H is a function of negative pore water pressures (soil water tension) and, hence, equation (3) is nonlinear. Depending on the orientation of the x and y coordinates, the temporal term in equation (3) may be zero or a function of the total saturated thickness multiplied by a storage coefficient. The moisture sink term for a freezing soil accounts for the conversion of liquid water to ice; i.e.,

$$S=L\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \quad (4)$$

One must consider three general regions in a freezing or thawing soil to apply equation (3). The unfrozen zone, which for instance may be between an advancing freezing front and a water table, will be adequately described by equation (3) assuming moisture movement is by connected liquid water films (in this case $S=0$). In the freezing zone, a zone of finite width depending on material type and perhaps other factors, equation (3) may apply and $S \neq 0$. A major difficulty in applying equation (3) in a freezing region is determining the hydraulic conductivity parameter. Nakano, et al. [24] has demonstrated that the presence of ice in soil pores significantly affects the transport of water in soils. The hydraulic conductivity of partially frozen soil is much less than for unfrozen soil. Jame [21] and Taylor and Luthin [28] using data developed by Jame, indicated that unfrozen hydraulic conductivity had to be reduced in a freezing zone in order to adequately model the thermal and soil moisture regime of freezing horizontal columns. They assumed a phenomenological relationship of the form

$$K=K_H 10^{-E\theta_i} \quad (5)$$

where K = freezing soil hydraulic conductivity, K_H = unfrozen soil hydraulic conductivity, θ_i = volumetric ice content, and E = a calibration factor such that $E\theta_i \geq 0$. For fully frozen soil, it is generally believed that unfrozen water may move as liquid water films primarily in response to hydraulic gradients. In this case it may be assumed that $S = 0$. Whether this flow may be represented by Darcy's law is not established nor are there definitive data available on frozen soil hydraulic conductivity.

Because unsaturated soils are common to freezing and thawing problems, some means of relating unfrozen water content and total hydraulic head or pore pressure is required. This is accomplished by developing the usual soil moisture characteristics which for clays and silts is hysteretic. For convenience, it is usually desirable for modeling purposes to assume that this relationship is single valued, removing the problem of incorporating memory in models. Further, it is convenient to express this experimental relationship as some form of mathematical function. While numerous such relationships have been proposed, it was found that Gardner's [7] relationship fits many soils of general interest, i.e., the so-called frost-susceptible soils. Water content is related to pore pressure as follows:

$$\theta=\frac{\theta_o}{A|u|^n+1} \quad (6)$$

A and n = regression fit coefficients which depend on soil type, density, and other factors.

Model Development

Since 1979 the U.S. Army, Cold Regions Research and Engineering Laboratory (Guymon, et al. [9]) has been developing a one-dimensional model of frost heave, thaw consolidation and thaw weakening, applicable to roadways and airfields. Guymon, et al. [12] and Berg, et al. [4] presented the concepts of a modeling approach and early verification and sensitivity results. Subsequently, Guymon, et al. [10] presented additional verification results, and Hromadka, et al. [17] presented a detailed evaluation of model sensitivity to the choice of numerical analog. Guymon, et al. [11] evaluate parameter sensitivity and develop a probabilistic model which is cascaded with the deterministic one-dimensional model. Finally, Hromadka, et al. [16] present data on freezing of an airfield embankment and results from the two-dimensional model discussed herein. Because the foregoing cited references explain modeling concepts in detail, only a brief discussion of the two-dimensional model will be presented in the forthcoming. Model assumptions are as follows:

1 Moisture flow occurs in unfrozen zones by liquid water films driven by hydraulic gradients and may be estimated by Darcy's law.

2 Moisture flow in frozen zones is negligible.

3 Sensible heat transport in all zones is governed by the heat transport equation.

4 Phase change effects may be decoupled from the governing transport equations and approximated as an isothermal freezing process.

5 Unfrozen zones are nondeformable, and in freezing zones or frozen zones, deformation is due to ice segregation or lens thawing only.

6 Soil-water pore pressures in freezing zones are governed by an unfrozen water content factor determined from soil freezing characteristics.

7 Hysteresis is not present.

8 Salt exclusion processes are negligible.

9 Constant parameters (e.g., porosity) remain constant with respect to time; i.e., freeze-thaw cycles do not modify parameters.

Nomenclature

A_k, m = parameters describing unsaturated hydraulic conductivity of unfrozen soil
A_w, n = parameters describing soil moisture characteristics for unfrozen soil
C_m = volumetric thermal conductivity
C_w = volumetric heat capacity of water
E = calibration parameter
K_H = hydraulic conductivity for unfrozen soil
K_o = saturated hydraulic conductivity of unfrozen soil
K_T = thermal conductivity
L = volumetric latent heat of fusion of soil water (assumed to equal bulk water)
n = normal coordinate
t = time
T = temperature
T_f = freezing point depression of water
u = pore water pressure expressed as hydraulic head
u_o = overburden pressure expressed as equivalent hydraulic head
v_x, v_y = velocity flux computed from Darcy's law
x, y = Cartesian coordinates
ϕ = total hydraulic head
ρ_i = density of ice
ρ_w = density of water
θ_i = volumetric ice content of soil
θ_n = unfrozen water content factor for frozen soil
θ_o = porosity of soil
θ_u = volumetric unfrozen water content of soil

10 Freezing and thawing processes in a two-dimensional medium occur in such a way that there are no internal shear or stress forces developed between different zones.

11 Excess pore pressures from consolidation are negligible and overburden and surcharge effects are primarily responsible for modifying pore-water presures in freezing zones provided ice segregation is occurring. Otherwise the soil matrix supports overburden and surcharge pressures.

The equations for the two-dimensional model are summarized in Table 1. The moisture flow equation is solved using total head as the state variable rather than using pore-water pressure as is customary in much of the literature. The motivation for using total head is that the numerical analog "stiffness" matrix will be symmetrical, reducing computer memory requirements. The convective terms of the heat transport equation are approximated as a space-time average from a previous solution timestep so that these terms may be included in the numerical analog load vector term. Thus the numerical analog "stiffness" matrix will be symmetrical, reducing computer memory storage requirements.

Latent heat terms are typically handled by the so-called apparent heat capacity approach (Luikov [22]). While there are no theoretical problems in using this approach for models that incorporate heat transfer alone, Hromadka, et al. [18] show that for coupled problems, the use of the apparent heat capacity concept may lead to inconsistent models with undesirable restraints on parameters and solution discretization. For these reasons, an isothermal approach is used to approximate phase change effects (Hromadka, et al. [18]).

The algorithm is based on a simple control volume approach. A volume of freezing soil is not allowed to reach a subfreezing temperature until the latent heat of fusion of all available water for freezing in a control volume of soil is exhausted. However, if this procedure is used for a large region of soil, it is difficult to determine the location of the freezing or thawing isotherm. Because there is a large difference in mechanical strength properties between frozen and unfrozen soil, the locations of the freezing isotherm is important and the freezing or thawing isotherm must be relatively precisely defined. This is done by using a pseudo apparent heat capacity approach. The numerical analog mass matrix diagonal terms are weighted so that phase volumes are undergoing soil water phase change.

DeVries [5] relationship given by equation (2) is used to compute the heat capacity and thermal conductivity of the soil-water-air-ice mixture. Solution of equation (3) requires that the relationship between pore-water pressure and water content be known where the temporal term is replaced by the term $[\partial\theta_u/\partial u]$ $[\partial\theta/\partial t]$. The partial, $\partial\theta_u/\partial u$, is determined from equation (6). Additionally, this same relationship is used to relate hydraulic conductivity to pore-water pressure; i.e.,

$$K_H(u) = \frac{K_o}{A_k\,|u|^m + 1} \tag{7}$$

where A_k and m are parameters for a given soil. Hydraulic conductivity in freezing zones is estimated by the phenomenological relationship given by equation (5).

Pore-water pressures at freezing fronts, which largely determine the hydraulic gradient toward a freezing front, are determined by

$$u = u(\theta_n) \tag{8}$$

where θ_n = a constant unfrozen water content factor. Although equation (8) is dependent on temperature (Anderson, et al. [1]) and perhaps pressure, a constant value is used in the current version of the model. If ice segregation is not occurring, the overburden and surcharge are assumed to be supported by the soil matrix. If ice segregation is occurring, then negative pore-water pressures given by equation (8) are modified by adding overburden and surcharge pressures u_o (expressed as equivalent hydraulic head), to equation (8); i.e.,

$$u_f = u(\theta_n) + u_o \tag{9}$$

where u_f is the pressure near the ice lens-water interface. The water film on the ice lens is presumed to support the total overburden and surcharge weight. While excess pore pressures, u_e, may be added to equation (9), we have found that these pressures are very small for frost-susceptible silts and sandy silts and may be conveniently neglected. Equation (9) simulates the physical processes that are assumed to occur at ice segregation fronts. Overburden and surcharge stresses reduce negative pore-water pressures given by equation (8), and thus reduce hydraulic gradients and moisture flow toward ice segregation fronts.

Auxiliary equations are required for boundary and initial conditions in order to solve the problem. Required initial conditions are soil-water temperatures, ice content, and pore-water pressures. While any type of boundary conditions may be incorporated (e.g., a soil surface heat budget simulator), we have generally used a functional relationship for soil surface temperatures based upon the U.S. Army, Corps of Engineers n-factor approach (Berg [3]). For pore pressure boundary conditions, the model assumes no moisture flux at frozen boundaries and uses a specified time-varying pore-water pressure at other boundaries.

Parameters required in the multi-parameter model are summarized in Table 2. Other required parameters such as the latent heat of fusion of water, heat capacities of ice and water, and density of water are taken from standard thermodynamic tables.

Numerical Analog

The nodal domain integration method is used to solve the heat and moisture transport equations given in Table 1. This method has been presented in detail by Hromadka, et al. [19] and elsewhere and will only be briefly reviewed here.

The solution domain is discretized into finite elements, similar to the finite element procedure. For this model, triangular elements are used where the state variable is represented by linear trial functions. Depending upon how one defines a subdomain of integration for the partial differential operator, various numerical analogs may be developed; e.g., Galerkin finite element analog. Hromadka, et al. [19] showed that an infinity of numerical analogs exist where the uncoupled linearized heat transport or moisture transport equations, given in Table 1, may be represented by the element matrix equation of the form

$$\underline{K}^e \phi_j + \underline{P}^e(\eta)\,\dot{\phi}_j = \{0\} \tag{10}$$

Equation (10) is for the moisture transport equation and where $\underline{K}^e$ = a symmetric banded conduction matrix identical to the usual stiffness matrix for linear trial function triangular elements, $\underline{P}^e(\eta)$ = a symmetric banded capacitance matrix as a function of a mass weighting factor η, ϕ_j = a vector of unknown state variables at nodal points, and $\dot{\phi}_j$ = a vector of the time derivative of the unknown state variables. Matrix $\underline{P}^e(\eta)$ is given by

$$\underline{P}^e(\eta) = \frac{(\partial\theta_u/\partial u)^e A^e}{3(\eta+2)} \begin{bmatrix} \eta & 1 & 1 \\ 1 & \eta & 1 \\ 1 & 1 & \eta \end{bmatrix} \tag{11}$$

where e = a particular triangular element and A^e = the element area. When η = 2, the usual Galerkin finite element formulation is obtained for a linear trial function approximation. When η = 22/7 (approximately π), a subdomain integration formulation is obtained. When $\eta \rightarrow \infty$, an integrated finite difference formulation is obtained.

Table 1 Deterministic model equations[a]

Soil region	Energy state	Liquid moisture transport	Phase change	Sensible heat transport
Freezing surface boundary		$\partial\phi/\partial n = 0$		$T_u = T(t) \le T_f$
Frozen	$T < T_f$ $u = u(\theta_n + \delta\sigma/\gamma)$ $\delta = \begin{cases} 1, & \theta_i \ge \theta_o - \theta_n \\ 0 & \theta_i < \theta_o - \theta_n \end{cases}$	$\nabla\phi = 0$	$\dfrac{\partial\theta_i}{\partial t} = 0$	$K_T \nabla^2 T = C_m \dfrac{\partial T}{\partial t}$
Freezing or thawing	$T = T_f$ $u(\theta_n) < u < 0$	$\dfrac{\partial\theta_u}{\partial t} = K_H \nabla^2 \phi$	$-\dfrac{\rho_i}{\rho_w}\dfrac{\partial\theta_i}{\partial t}$ $L\dfrac{\rho_i}{\rho_w}\dfrac{\partial\theta_i}{\partial t}$	$+K_T \nabla^2 T = C_m \dfrac{\partial T}{\partial t} + C_w\left(v_x \dfrac{\partial T}{\partial x} + v_y \dfrac{\partial T}{\partial y}\right)$
Unfrozen	$T > T_f$ $u(\theta_n) < u < u_L$	$\dfrac{\partial\theta_u}{\partial t} = K_H \nabla^2 \phi$	$\dfrac{\partial\theta_i}{\partial t} = 0$	$K_T \nabla^2 T = C_m \dfrac{\partial T}{\partial t} + C_w\left(v_x \dfrac{\partial T}{\partial x} + v_y \dfrac{\partial T}{\partial y}\right)$
Unfrozen surface boundary		$u_L = u(t)$		$T_L = T(t) > T_f$

[a] See nomenclature for a definition of symbols. Ancillary relationships are described in text.

Table 2 Parameters required for the two-dimensional frost heave model

Parameter	Description
n, A_w	characterize volumetric water content versus pore water pressure relationship for unfrozen soil
K_s	saturated hydraulic conductivity for unfrozen soil
m, A_K	characterize soil pore water pressure versus hydraulic conductivity relationship for unsaturated unfrozen soil
E	corrects unfrozen unsaturated hydraulic conductivity for hydraulic conductivity in partial frozen soil in freezing zone (a calibration factor)
θ_o	soil porosity
θ_n	unfrozen water content factor in freezing or frozen soil (minimum soil water content assumed to coexist with ice)
K_s	thermal conductivity of soil
ρ	density of soil
T_f	freezing point depression of soil water

Hromadka and Guymon [16] show that η may be a function of time and that the best choice of a η depends upon the nature of the problem studied. For example, where sharp wetting fronts may occur in a porous media seepage problem, the integrated finite difference formulation generally results in less relative error than the Galerkin finite element formulation.

The derivation of equation (10) requires that parameters be held constant in each subdomain or element; however, parameters may vary from subdomain to subdomain. Additionally, each nonlinear partial differential equation is linearized where parameters are held constant for a small time interval. This procedure is in lieu of an iterative scheme to account for nonlinear parameters. This procedure is often valid for soil problems since the soil water phase change effects result in a damped system of partial differential equations.

For the entire solution domain, matrix equation (10) is assembled in an appropriate manner to form the system matrix equation

$$\underline{K}^e \phi_j + \underline{P}(\eta)\dot{\phi}_j = \{0\} \tag{12}$$

where $\underline{K}$ and $\underline{P}(\eta)$ are square banded symmetric positive definite matrices, ϕ_j and $\dot{\phi}_j$ are, respectively, unknown nodal state variables and their temporal derivative, and F_j is a vector of known boundary conditions.

Solution of equation (12) is by a fully implicit approach which is required to solve problems where a free water surface exists within the solution domain (Neuman, [25]); i.e., equation (12) becomes

$$(\underline{K} + \underline{P}(\eta)/\Delta t)\phi_j^{t+\Delta t} - \underline{P}\phi_j^t/\Delta t = F_j^{t+\Delta t} \tag{13}$$

where Δt = an appropriate timestep interval.

The solution procedure for both uncoupled equations of state (given in Table 1) is to use equation (13) to solve for the state variables ϕ and T. At specified intervals Δt^*, where $\Delta t^* \ge \Delta t$, all nonlinear parameters are recomputed, using the necessary ancillary relationships discussed previously, and the system matrices are "updated." Ice contents and secondary variables are computed at this time. Latent heat effects and total lumped ice segregation quantities are evaluated at each Δt timestep. Boundary condition information is updated at each Δt^* interval.

The numerical model, FROST2B, is coded in FORTRAN IV for use on mini-class computers. The model includes a front-end humanized, interactive data preparation program, PROTOØ, and a color graphics output program, ROAD. The model is coded in a modular form allowing easy modifications to a general class of problems. This version is for an arbitrary cross section and does not permit the

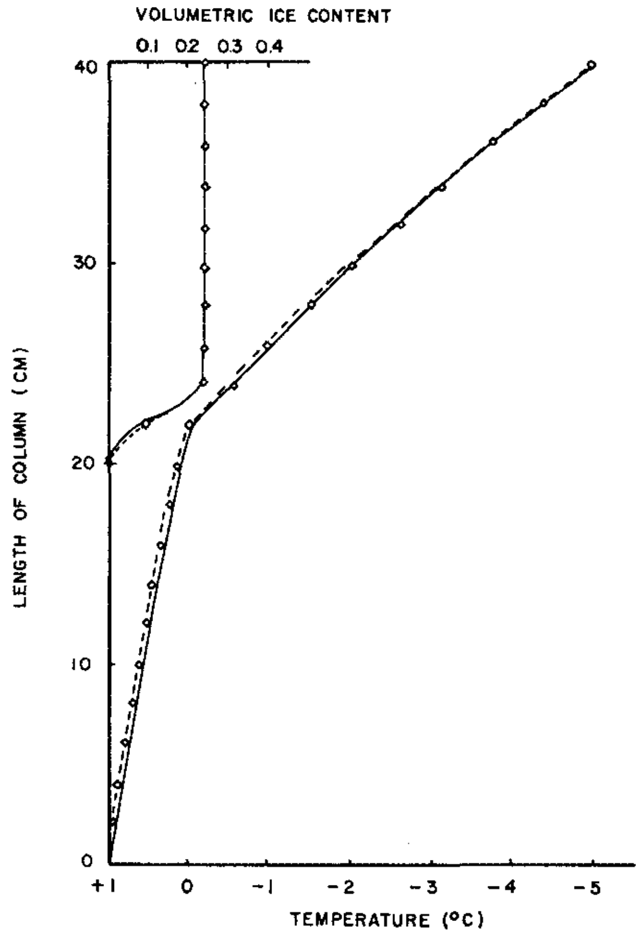

Fig. 1 Comparison of one and two-dimensional model solutions at the end of 18 hr of freezing an initially unfrozen vertical column of moist soil. Solid line is for 2-D model solution and dashed line is for 1-D model solution.

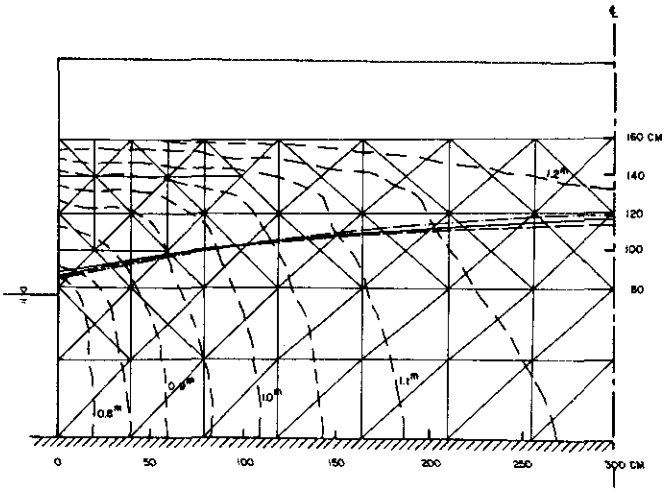

Fig. 2 Free water surface positions 1 hr after lowering ditch water level for an unfrozen isothermal sandy soil. Finite element grid is shown and equipotential lines are dashed. The water surface line is solid for experimental, dot-dashed for Vauclin, et al. [30] solution, and dashed for 2-D model solution.

calculation of frost heave. A second version (*D*) is for a roadway embankment and permits frost heave calculations. FROST2D includes a mesh generator in order to keep track of differential frost heave at the roadway and embankment surface. All versions are capable of dealing with a layered soil profile.

Model Verification

Because of the nonlinear nature of the coupled problem and the model selected to represent frost heave, the only available method of verification is comparing model results to prototype data which may be a physical laboratory model or field situation. Even this approach is not entirely satisfactory because of uncertainty associated with boundary and initial conditions and parameters that arise in the model. While most of the parameters imbedded in the model have some presumed physical meaning and can be evaluated in a laboratory, the E-parameter (see equation (5)) can only be determined by calibration of the model. Sufficient calibration efforts may suggest a way of predetermining E in the future.

Linearized decoupled problems may be solved analytically to determine the accuracy of models. Because spatial and temporal discretiation interact with model errors, these problems need to be studied at the same time. Most analytical solutions of freezing soil or bulk water are for one-dimensional columns. Thus, accuracy of a two-dimensional model may be studied by solving column problems oriented in the x direction and then in the y direciton. This procedure was followed through several tests for both unsaturated soil moisture transport and heat transport, with and without phase change. For heat transport alone, errors in the position of isotherms and particularly the freezing isotherm were less than 8 percent for relatively fine spatial discretization and fairly large timesteps. Errors could be reduced to less than 3 percent for smaller timesteps. Unsaturated soil moisture transport in a vertical column was evaluated by comparing to a quasi-analytical solution as discussed in Guymon and Luthin [13]. Close agreement was obtained depending primarily on how frequent nonlinear hydraulic conductivity is updated in the model.

Additionally, the two-dimensional model was compared to one-dimensional model solutions and one-dimensional laboratory soil column tests. The one-dimensional model has been extensively verified against soil column data and field data (Guymon, et al. [10]; Guymon, et al. [11]; and Guymon, et al. [9]). Figure 1 shows an example comparison for a coupled heat and moisture transport problem involving Fairbanks silt. In this example, the column top was subjected to a -5°C temperature at time zero and the column bottom was maintained at the initial condition temperature of 1°C. A water table was maintained at the column bottom.

The two-dimensional model was tested against a number of isothermal unfrozen soil dam problems involving unsteady seepage with a free water surface. Figure 2 shows an example comparison of a model solution to experimental data and a solution of Vauclin; et al. [30] for the phreatic surface for an unsteady ditch drainage problem. Experimental data and theoretical solutions compared favorably. Figure 3 shows an example of a dam seepage problem where a dynamic model solution is compared to Uginchus' [29] steady-state theoretical solution.

Two-dimensional verification of the heat transport model has included comparison with field data for a laboratory sand tank. In the first case model solutions were compared to sparce thermistor data for the summer and various winter times of the year at eight locations in a roadway located west of Fairbanks, Alaska. Boundary conditions were only approximately known and thermal properties of the different embankment materials were approximated from textural classification information. A second comparison was made with freezing data for the Deadhorse runway, Alaska; excellent results were obtained, Guymon, et al. [9]. In all cases,

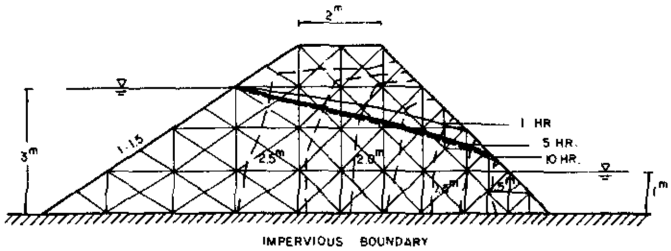

Fig. 3 Simulated transient positions of a water table (solid line) in an earth dam. Uginchus [29] steady-state solution of water table position is shown as dashed lines. The finite element grid is shown and equipotential lines (dashed) are shown.

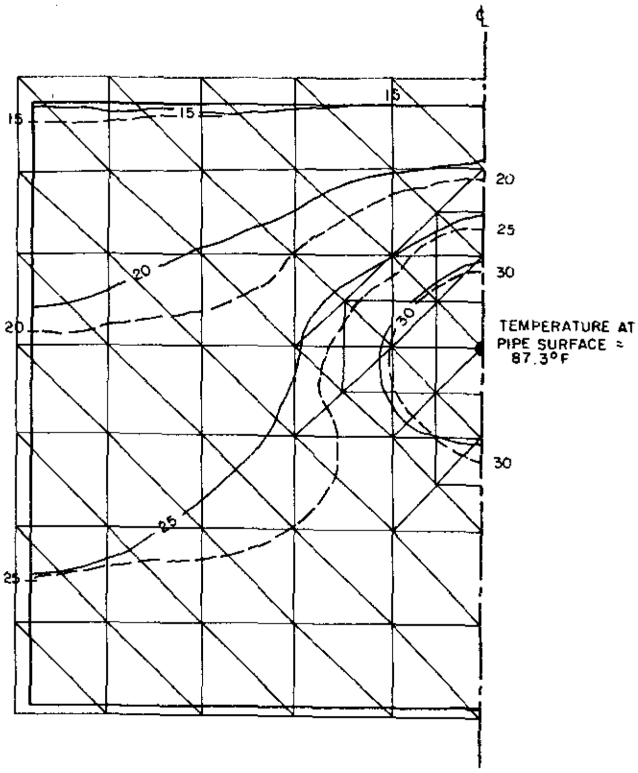

Fig. 4 Comparison of experimental (solid lines) and simulated (dashed lines) soil temperatures for soil tank model

estimated temperatures were within 1°C of measured temperatures. The position of the freezing isotherm was accurately estimated, errors being only a fraction of a foot in most cases.

The sand tank model consisted of a 3.92-m wide by 1.28-m deep tank of sandy silt that is over 4-m long to simulate two-dimensional thawing around a buried small diameter hot pipe. The embankment is initially frozen from the surface down by means of cold plates. Sides and bottoms are insulated to minimize heat loss. The upper boundary condition and pipe temperature boundary conditions are known. Side and bottom boundary conditions are assumed to be zero heat flux. Soil thermal parameters and initial soil ice contents were assumed. Because of symmetry only half the tank was analyzed, where at the pipe centerline, zero heat flux in the x direction was assumed. A comparison of modeled and measured soil temperatures after one-day of initiating hot fluid flow in the buried pipe is shown in Fig. 4. As time progresses, the solution at the bottom of the tank deviated somewhat from measured temperatures, primarily because of inaccurate representation of the bottom temperature and heat flux conditions.

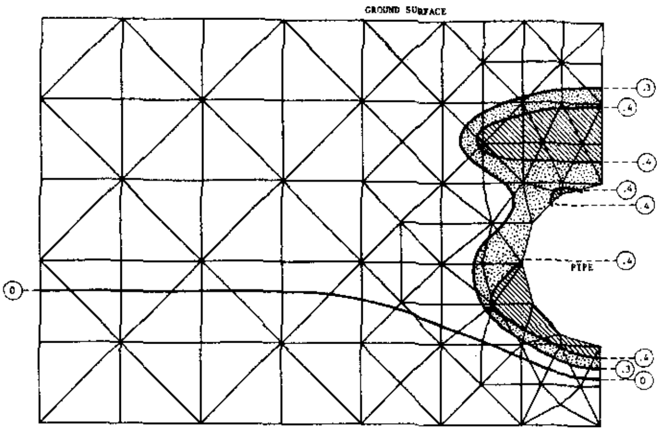

Fig. 5 Simulated soil ice content after 6-mo of freezing from a buried chilled pipeline. Ice contents are shown by circled numbers. The finite element grid is shown.

Verification of the two-dimensional model for uncoupled moisture transport alone or uncoupled heat transport alone has demonstrated that these two important components of the overall model are accurately modeled. In particular, the isothermal freezing approach provides a relatively accurate and economical prediction of freezing and thawing phenomena. Rather large spatial discretization may be used. Unfortunately, we do not have a good data set for a two-dimensional frost heave prototype situation in order to further verify the model. However, one-dimensional data is available as described by Ingersoll and Berg [20]. One-dimensional solutions using the two-dimensional model are identical to results achieved by the one-dimensional model which can accurately simulate unrestrained and restrained frost heave.

Evaluation of parameter errors and numerical analog errors have been extensively dealt with previously for the one-dimensional model (Guymon, et al. [11] and Hromadka, et al. [17]). Similar to the one-dimensional model, the two-dimensional model is sensitive primarily to hydraulic conductivity parameters. Some success has been obtained in calibrating the E-parameter using split record tests. This, is, the E-parameter may be calibrated using a 1-yr sequence of data, and without modification of parameters, following a one-year sequence of frost heave data may be reliably evaluated. Thermal parameters may be approximated without large error in results. This is primarily due to the fact that latent heat effects largely dominate the thermal regime of a freezing or thawing soil. It is emphasized that the model has been primarily tested against frost susceptible silts and dirty gravels. Additionally, the method of dealing with surcharge and overburden appears to only apply for relatively low overburden pressures (i.e., $u_o < 60$ kPa).

Example Applications

The first problem considered is a buried chilled pipeline, 1.2 m in diameter. The problem domain is 4.3 m, by 3.1 m in the horizontal and vertical dimensions, respectively. The soil is assumed to be a homogeneous Fairbanks silt with an initial homogeneous temperature of 0.5°C and pore pressure head of −70 cm. Initial ice content is assumed to be zero throughout the soil region. The pipeline is assumed to be buried 1.2 m below the ground surface. The groundwater table effects are modeled by assuming a constant pore-pressure head of −70 cm 0.6 m below the bottom of the pipe. The solution is assumed to be symmetrical about the vertical centerline of the pipe. A constant thermal boundary condition of −2°C and .5°C is assumed at the top and bottom of the study region, respectively. The pipe surface is assumed to

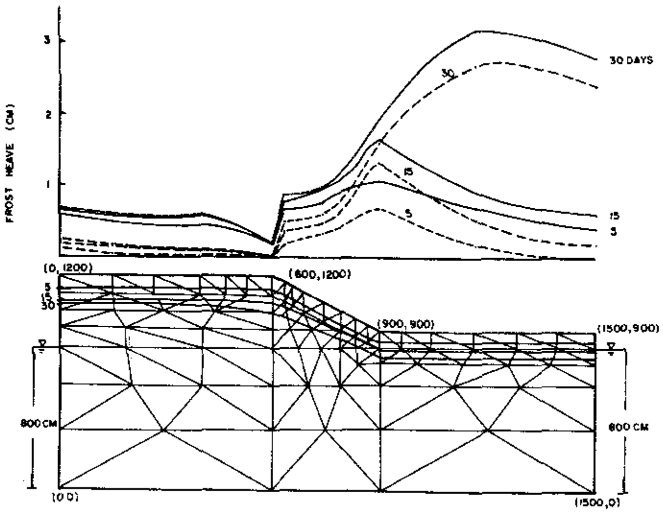

Fig. 6 Simulated frost heave in a roadway embankment for indicated days after surface freezing started. Coordinates shown are cm. The position of the freezing isotherm is shown as a solid line. The computer generated finite element grid is also shown. Dashed lines are for a 1.2-kPa surcharge. Solid lines are for zero surcharge.

remain at a constant temperature of −.5°C. Overburden effects are not considered, and the total simulation is for 6 mo. Estimated ice contents at the end of a 6-mo period are plotted in Fig. 5. Cross-hatched areas indicate regions in which ice contents exceed the assumed soil porosity and ice-segregation may be occurring. Dotted areas indicate where ice contents are approaching the assumed porosity.

Results of the two-dimensional model are somewhat verified by assuming a one-dimensional problem at the vertical soil region furthest from the pipe. Comparison of one-dimensional model results to the pore pressures and temperatures computed by the two-dimensional version of the model were found to be in close agreement.

Frost heave is not estimated in this example because of the unknown effects of overburden and pipe interaction with the freezing soil. Also, the interaction between soil regions where ice segregation may be taking place and other soil regions is not known. For complicated geometry such as is considered in this example, only the heat transport and moisture transport aspects of the model are valid. However, the tendency for ice segregation to occur can be qualitatively evaluated with the model.

A second example is a roadway embankment where it is assumed that soil interaction problems are minor and frost heave may be estimated. Figure 6 shows the soil solution domain for half of a roadway and the simulated frost heave at two different overburden values and for various days after initiation of freezing. Parameters for Fairbanks silt were assumed. A horizontal water table exists in the embankment as shown. Soil surface temperatures were assumed to be a constant −20°C and soil bottom temperatures were assumed to be a constant 1°C. Horizontal transport of moisture or heat at the solution domain sides was assumed to be zero.

As an approximate check, a one-dimensional solution, using the one-dimensional model, was compared to two-dimensional solution results at the roadway centerline and the right side of the solution domain. Centerline solutions compared very closely while right-side solutions using the one-dimensional model were about 0.5 cm of heave greater than estimated with the two-dimensional model. However, the solutions were tending to converge after 30 days. The reason for this discrepancy may be that relatively large elements were used in the two-dimensional model. The results appear to be conceptually correct. The water table position strongly influences simulated heave. A rather small surcharge restrains frost heave somewhat. At the roadway edge, frost heave is reduced due to more rapid freezing caused by the surface geometry. At the embankment toe, frost heave is larger than for other areas for times up to 15 days because surface geometry results in less heat extraction.

Discussion

A two-dimensional model of coupled heat and moisture transport in freezing or thawing soil is presented. For simple but useful geometries, frost heave or thaw consolidation may be estimated. The model is based upon well known transport equations to estimate heat and water transport for a soil freezing or thawing problem. Because the model is based upon incomplete theory for freezing zones, phenomenological relationships are also employed.

This results in a major limitation of the model since one phenomenological parameter (E) must be determined from laboratory freezing experiments or field data on a freezing soil. Guymon, et al. [9] found that the E parameter may be reasonably evaluated for field cases provided a number of freeze-thaw cycles are included in the data.

Other parameters required for the model, such as soil porosity and density, can be routinely determined in the laboratory. However, hydraulic parameters such as unsaturated hydraulic conductivity and the unfrozen water content factor, θ_n, require specially equipped laboratories to determine them. Although there are such laboratories, their need is somewhat of a limitation to the routine employment of the model by the geotechnical cadre. In an effort to minimize this problem the U.S. Army-CRREL is attempting to analyze a large number of soils with the hope that this data may be of value in estimating sophisticated parameters without conducting costly tests. Some of this data is summarized in Guymon, et al. [9].

Conclusions

A multiparameter two-dimensional model of coupled heat and moisture transport for frost heaving soils is feasible based upon the concepts presented herein. However, because of a lack of detailed knowledge concerning forces between interacting freezing and unfrozen soil masses, this model is limited to simple geometries. Models that would be applicable to more complex geometries will require substantially more research. The model proposed here can accurately predict frost penetration and heave (for simple geometries) provided laboratory or field data are available for calibration of sensitive parameters, primarily hydraulic conductivity. In the examples presented herein, we have chosen to calibrate a phenomenological parameter that reduces unfrozen hydraulic conductivity in freezing zones.

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