A two-dimensional dam-break flood plain model

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A simple two-dimensional dam-break model is developed for flood plain study purposes. Both a
finite difference grid and an irregular triangle element integrated finite difference formulation are
presented. The governing flow equations are approximately solved as a diffusion model coupled to
the equation of continuity. Application of the model to a hypothetical dam-break study indicates
that the approach can be used to predict a two-dimensional dam-break flood plain over a broad,
flat plain more accurately than a one-dimensional model, especially when the flow can break-out of
the main channel and then return to the channel at other downstream reaches.

Key Words: Unsteady flow, dam-break analysis, two-dimensional hydrodynamic model

INTRODUCTION

The probable flooding damages which may occur due to a
dam failure is of concern to many civil engineers and
planners. Not only do such studies provide a source of
information for insurance and flood control studies, but
the actual planning process for the construction of a dam
site can be modified by the results of such a predictive
analysis.

Generally, such dam-break studies can be completed by
either scaled hydraulic models or by use of computer
simulation. The most cost effective approach is to
approximately solve the governing flow equations of
momentum and continuity by computer simulation,
where several situations can be considered by a fraction of
the cost of a scaled prototype model.

For water courses in which flows can be classified as
one-dimensional, several models are presented and
verified in the literature. One widely used one-
dimensional model which is utilized in this study is the K-
634 model version developed by Land.

However for flow conditions which are truly two-
dimensional, such as occur when massive dam-break
flows exceed the water course channel capacities and
excess flows break out and travel away from the water
course, then a one-dimensional approximation may be
inappropriate.

In this paper, a simple two-dimensional dam-break
model is developed and applied to a hypothetical dam
failure situation where channel break out flows are a
major factor in the determination of a flood plain. The
model is based on a diffusion approach where gravity,
friction, and pressure forces are assumed to dominate the
flow equations. Such an approach has been used earlier by
Xanthopoulos and Koutitas in the prediction of dam-
break flood plains in Greece. In those studies, good results
were also obtained in the use of the two-dimensional
model in predicting one-dimensional flow quantities. The
diffusion model has also been used to successfully model
one-dimensional channel flows. In another paper, Hromadka considers a one-dimensional diffusion model
and concluded that for low-to-moderate velocity flow
regimes (i.e., less than approximately 25 ft/sec), the
diffusion model is a reasonable approximation of the full
dynamic wave formulation.

In this paper, two versions of the two-dimensional
diffusion model are presented. A finite difference grid
model is developed which equates each cell-centered node
to a function of the four neighboring cell nodal points.
Additionally, the finite difference model is extended to an
integrated finite difference analog based on irregular
triangles using a nodal domain integration (NDI) control
volume definition for each node. Both models are used to
generate hypothetical flood plains, and these results are
compared to a similar study based on the one-
dimensional K-634 model.

A comparison and analysis of model results shows that
the two-dimensional diffusion approach provides a more
reasonable representation of two-dimensional flow effects
than does a fully dynamic one-dimensional model, and
that the diffusion model affords an easy-to-use predictive
tool for the estimation of dam-break flooding depths over
two-dimensional flood plains.

STUDY AREA DESCRIPTION

The Owens River drains the rugged eastern slopes of the
Sierra Nevada Range, the Benton Range, and the western
slopes of White Mountains located in Inyo County,
California (Fig. 1). The head waters of the Owens River
flows into Lake Crowley (formed by the Long Valley
dam). The dam is an earth-fill type, 126 ft in height with a
crest length of 595 ft, and a total storage capacity of
183,465 acre-feet of water. Downstream from Long Valley
Dam, the Owens River is constrained in an incised canyon.
of volcanic Bishop tuff formation. The river flows through the gorge to the canyon mouth, seventeen miles downstream from the dam. The river encounters several hydroelectric power plants in reaching the canyon mouth. However in this study, the effects of these obstructions in a dam-break analysis are assumed minor due to the relative magnitude of the flood volume compared to the volume afforded by each power plant.

Through the first several miles downstream from the canyon mouth, the river meanders through alluvial fans with a slope of 50 ft per mile. Four miles downstream from the canyon mouth, the river enters Owens Valley which is a flat alluvial valley with a slope of only 10 ft per mile. (The basin elevations range from 4300 ft to 3800 ft). The river curves around the City of Bishop about 17 miles downstream from the canyon mouth, and then meanders through Owens Valley for another 40 miles until it reaches Tinemaka Reservoir. Figures 1 and 2 illustrate the study location and vicinity.

One-dimensional analysis approach

In this section, the one-dimensional dynamic wave, implicit dam-break model K-634 developed by Land	extsuperscript{1,2} is used to investigate a hypothetical dam failure and resulting flooding along the Owens River.

The K-634 model routes a flood hydrograph through a reservoir with a water level initially below the position where a breach in the retaining dam is assumed to occur. The routing of this hydrograph is accomplished by one of two procedures; namely, a hydrologic method (storage-continuity), or a hydraulic method such as discussed in Land	extsuperscript{1,2}. When the water level in the reservoir reaches a preselected level, a breach in the dam is assumed to begin. The breach begins with a zero width, and widens and deepens during a specified time of failure. At the end of the failure, the breach is fully developed and is assumed to remain constant for the remainder of the simulation. The breach flow rates are computed based on a trapezoidal-shaped critical flow area. The outflow hydrograph at the dam is computed as the sum of the initial flow through dam structures and the flow through the breach.

The dam-break flood is then mathematically routed downstream by solving the one-dimensional Saint Venant flow equations using a nonlinear implicit finite-difference algorithm.

One-dimensional model data requirements

The K-634 data requirements fall into three categories.

1. In the reservoir: surface area versus elevation tables, or channel geometry and roughness; (2) at the dam: breach shape, duration of breach development, dam outlet structures stage-outflow ratings, and water-surface elevation when failure begins to occur; and (3) in the stream: channel geometry, roughness and state of flow (e.g. subcritical or supercritical). The upstream boundary condition for routing the flood downstream is the dam-break outflow hydrograph. The downstream boundary
condition is a dynamic stage-discharge relation which is computed from the full dynamic flow equations.

The one-dimensional model grid schematization is shown in Fig. 2. The channel geometry of the reach of river was defined by field-surveyed cross sections where, for each section, a maximum of eight elevation versus top width data pairs were specified. Due to the relatively flat topography perpendicular to the main channel (see Figs. 2 and 6), the uppermost portions of the channel cross sections were assumed to have a mild gradient. This modeling assumption results in a false confinement of channel flows due to a fictitious boundary being assumed on both sides of the channel. Consequently, predicted flood plains will be approximate at best, and requires additional interpretation. The effects of this modeling assumption will be further discussed in a subsequent section.

Mathematical formulation for two-dimensional model

The Saint Venant equations for two-dimensional flow may be written as

\[ \frac{\partial Q_x}{\partial t} + \frac{\partial Q_y}{\partial x} + \frac{gA_x}{S_{fs}} \frac{\partial H}{\partial x} + gA_x \left( S_{fr} + \frac{\partial H}{\partial x} \right) = 0 \]  

(1)

\[ \frac{\partial Q_y}{\partial t} + \frac{\partial Q_x}{\partial y} + \frac{gA_y}{S_{fs}} \frac{\partial H}{\partial y} + gA_y \left( S_{fr} + \frac{\partial H}{\partial y} \right) = 0 \]  

(2)

\[ \frac{\partial Q_x}{\partial t} + \frac{\partial Q_y}{\partial y} + \frac{gA_y}{S_{fs}} \frac{\partial H}{\partial x} - gA_y \left( S_{fr} + \frac{\partial H}{\partial y} \right) = 0 \]  

(3)

where \( Q_x, Q_y \) are flow rates in \( x, y \)-directions; \( g, A_x, A_y \) flow rates, length in \( x, y \)-directions; \( A_x, A_y \) directional cross-section areas; \( S_{fs}, S_{fr} \) directional friction slopes; \( x, y \), spatial and temporal coordinates; \( g \), gravity; \( H \), water surface elevation; where, for a grid discretization (Fig. 3), \( Q_x \) equals the product of \( q_y \) and the grid width, \( \delta \). The equation of continuity (equation (1)) is based on the assumption of constant fluid density with zero sources or sinks in the flow field. The \( x \)- and \( y \)-direction momentum relations (equations (2) and (3)) assume hydrostatic pressure distributions.

The local and convective acceleration momentum terms can be grouped together such that equations (1), (2) and (3) are rewritten as

\[ m_x + \left( S_{fr} + \frac{\partial H}{\partial x} \right) = 0, \quad z = x, y \]  

(4)

where \( m_x \) represents the sum of the first three terms in equations (2) and (3), and divided by \( gA_x \). Assuming the friction slope to be approximated by steady flow conditions, the Manning's equation in inch-pound units can be used to estimate

\[ Q_x = \frac{1.486}{n} A_x R_s^{1/3} S_{fs}^{1/2} \]  

(5)

where \( A_x, A_y \) are directional flow areas; \( R_s, R_x \) directional hydraulic radii. Equation (5) can be rewritten as

If the momentum term groupings were retained, equation (9) would be written as

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} + S = \frac{\partial H}{\partial t} A$$  

(10)

where \( A \) is the nodal control volume area and

$$S = \frac{\partial}{\partial x} (K_x m_x) + \frac{\partial}{\partial y} (K_y m_y)$$

and \( K_x, K_y \) are also functions of \( m_x, m_y \), respectively.

**Numerical model formulation (grid elements)**

For uniform grid elements, the numerical modeling approach used is the integrated finite difference version of the nodal domain integration (NDI) method. For grid elements, the NDI nodal equation is based on the usual nodal system shown in Fig. 3. Flow rates along the boundary \( \Gamma \) are estimated using a linear trial function assumption between nodal points. For a square grid of width \( \delta \)

$$Q_{\Gamma_x} = - (K_x |\Gamma_y|) (H_x - H_c)/\delta$$  

(11)

where

$$K_x |\Gamma_y| = \begin{cases} 1.486 (AR^{2/3}/n)^{1/2} |\Gamma_y| (H_x - H_c)/\delta & ; \delta > 0 \\ 0 & ; \delta \leq 0 \text{ or } |H_x - H_c| < 10^{-3} \end{cases}$$  

(12)

In equation (12), the terms \( A, R, \) and \( n \) are evaluated at the average flow depth defined by \( \bar{D} = (D_x + D_y)/2 \), and \( n = (n_x + n_y)/2 \). Additional, the denominator of \( K_x \) is checked such that \( K_x \) is set to zero if \( |H_x - H_c| \) is less than a tolerance such as \( 10^{-3} \) ft.

The model advances in time by an explicit approach

$$H^{t+1} = KH^t$$  

(13)

where the assumed dam-break flows are added to the specified input nodes at each timestep. After each timestep, the conduction parameters of equation (12) are re-evaluated, and the solution of equation (13) is continued. Using grid sizes with uniform lengths of one-half mile, the model advances in time by an explicit approach

$$Q_{\Gamma_x} = - (K_x |\Gamma_y|) (H_x - H_c)/\delta$$  

(14)

where \( K_x \) is evaluated similar to equation (12) at the triangle centroid \( (\xi, \eta) \), and \( |\Gamma_j| \) is the length of the NDI boundary projected on the -y-axis (see Fig. 5). Hromadka and Guymon show that the assemblage of NDI flow rate terms result in an element conduction matrix system \( KH + KH^{2/3} \) which is identical to the well-known Galerkin conduction matrix for the simplex element. Integrating the linear trial function on each nodal domain...
The NDI triangular element model requires nodal point coordinates and elevations, and averaged triangular element friction factors.

Both models include a critical depth (specified) boundary condition whereby flow depths may be assumed to correspond to critical flow conditions.

Finally, the inflow hydrograph may be specified at one or more nodal points. In this study, the form of this hydrograph was specified as the change in flow rate per 5-minute unit period. This time derivative of inflow is used to provide a smooth inflow hydrograph stepped according to the specified model time step.

CASE-STUDY RESULTS AND DISCUSSION

The K-634 model was initially applied to the steep canyon reach immediately downstream of the assumed dam-break. The modeling results indicated negligible attenuation of the flood wave peak and, consequently, subsequent studies of the two-dimensional plain were assumed to have the dam-break located at the downstream point of the canyon, neglecting the effects of the long canyon reach.

Applying the K-634 model to computing the two-dimensional flow was attempted by means of the one-dimensional nodal spacing shown in Fig. 2. Cross sections were obtained by field survey, and the elevation data used to construct nodal point flow-width versus stage diagrams for use in the K-634 model. A constant friction factor (Manning's) of 0.04 was assumed for study purposes. The assumed dam-break failure reached a peak flow rate of 420,000 cfs within one hour, and returned to zero flow 9.67 hours later. The resulting K-634 flood plain limits is shown in Fig. 6. As discussed in a previous section, a slight gradient was assumed for the topography perpendicular to the main channel. The motivation for specifying such a gradient was to limit the channel floodway section in order to approximately conserve the one-dimensional momentum equations. As a result of this assumption, fictitious channel sides are included in the K-634 model.
study which results in an artificial confinement of the flows. Thus a narrower flood plain is delineated (such as shown in Fig. 6) where the flood flows are falsely retained within a hypothetical channel confine. An examination of the flood depths given in Fig. 9 indicates that at the widest flood plain expanse of Fig. 6, the flood depth is about 6 feet, yet the flood plain is not delineated to expand southerly, but is modeled to terminate based on the assumed gradient of the topography towards the channel. Such complications in accommodating an expanding flood plain when using a one-dimensional model are obviously avoided by using a two-dimensional approach.

The two-dimensional diffusion models were applied to the total flood plain area using the grid and triangular discretizations shown in Figs. 7 and 8, respectively. The same inflow hydrograph produced by the K-634 model was used for both two-dimensional simulations. Again, the Manning’s friction factor of 0.04 was used. The resulting flood plain is shown in Fig. 7 for the 4-square-mile grid model. The 168 triangular element NID model resulted in a similar predicted flood plain (Fig. 8).

Comparisons of predicted maximum water elevations are shown in Fig. 9 which plots K-634 modeling results and the two-dimensional modeling results. The two approaches are comparable except at points shown as A and B in the figure. Point A corresponds to the predicted breakout of flows away from the Owens River channel with flows traveling southerly towards the City of Bishop. As discussed previously, the K-634 predicted flood depth corresponds to a flow depth of 6 feet (above natural ground) which is actually unconfined by the channel. The natural topography will not support such a flood depth and, consequently, there should be southerly breakout flows such as predicted by the two-dimensional models. With such breakout flows included, it is reasonable that the two-dimensional models would predict a lower flow depth at point A.

At point B, the K-634 model predicts a flood depth of approximately 2 feet less than the two-dimensional models. However at this location, the K-634 modeling results are based on cross-sections which traverse a 90-degree bend. In this case the K-634 model will overestimate the true channel storage, resulting in an underestimation of flow depths.

In comparing the various model predicted flood depths and delineated plains, it is seen that the two-dimensional diffusion model produced more reasonable predictions of the two-dimensional flood plain characteristics which are associated with broad, flat plains such as found at the study site than the one-dimensional model. The two-dimensional model affords approximation of channel bends, channel expansions and contractions, flow breakouts, and the general area of inundation. Additionally, the two-dimensional modeling approach allows for the inclusion of return flows (to the main channel) which result due to upstream channel breakout flows, and other two-dimensional flow effects without the need for special modeling accommodations which would be required when using a one-dimensional model.

Model sensitivity

The sensitivity of the overall modeling approach is illustrated for the case of K-634 by examining the three parameters of nodal spacing, Manning’s n, and dam-break time of failure. The base run values are a nodal spacing of 0.25 mile, Manning’s friction factor of 0.04, and a 1-hour dam-failure duration.
Fig. 7. Floodplain and grid layout for two-dimensional diffusion model

Fig. 8. Floodplain and triangle layout for two-dimensional diffusion model (108 nodes)
PREDICTED WATER SURFACE ELEVATIONS

Fig. 9. Comparison of modeled water surface elevations

Table 1. K-634 sensitivity analysis

<table>
<thead>
<tr>
<th>River Mile</th>
<th>2.0</th>
<th>8.4</th>
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</thead>
<tbody>
<tr>
<td>Node spacing</td>
<td>Maximum flowrate (cfs)</td>
<td>Maximum elevation</td>
</tr>
<tr>
<td>0.25 mi*</td>
<td>408.038</td>
<td>4208.80</td>
</tr>
<tr>
<td>0.30 mi</td>
<td>412.266</td>
<td>4208.98</td>
</tr>
<tr>
<td>0.40 mi</td>
<td>412.082</td>
<td>4209.11</td>
</tr>
<tr>
<td>0.50 mi</td>
<td>412.129</td>
<td>4209.12</td>
</tr>
<tr>
<td>Time of failure</td>
<td>Maximum flowrate (cfs)</td>
<td>Maximum elevation</td>
</tr>
<tr>
<td>0.50 hr</td>
<td>420.301</td>
<td>4209.04</td>
</tr>
<tr>
<td>1.0 hr*</td>
<td>408.038</td>
<td>4208.80</td>
</tr>
<tr>
<td>1.5 hr</td>
<td>400.352</td>
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<tr>
<td>1.75 hr</td>
<td>399.380</td>
<td>4208.64</td>
</tr>
<tr>
<td>2.0 hr</td>
<td>394.846</td>
<td>4208.56</td>
</tr>
<tr>
<td>Manning's n</td>
<td>Maximum flowrate (cfs)</td>
<td>Maximum elevation</td>
</tr>
<tr>
<td>0.04*</td>
<td>408.038</td>
<td>4208.80</td>
</tr>
<tr>
<td>0.045</td>
<td>411.221</td>
<td>4208.82</td>
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<tr>
<td>0.05</td>
<td>410.368</td>
<td>4210.74</td>
</tr>
</tbody>
</table>

* Base run used for flood plain study purposes

It can be seen in Table 1 that the most sensitive parameter is the friction factor, followed by the relatively insensitive parameters of time of failure and the nodal spacing. Similar sensitivity is found for the diffusion models.

CONCLUSIONS

A simple two-dimensional diffusion model was developed which can be based on either a finite-difference grid or a NDI irregular triangular element discretization. The keynes of the two-dimensional model is the diffusion version of the Saint Venant flow equations. Application of the model to the prediction of two-dimensional dam-break flood waves over broad, flat plains indicate that the diffusion model provides a significant advantage over the corresponding one-dimensional models in the study of flood plains such as occurs from a dam-break. The two-dimensional model is simple to use, requires readily available data, and does not need special modeling techniques to approximate two-dimensional flood flow effects.

REFERENCES


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