A two-dimensional dam-break flood plain model

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A simple two-dimensional dam-break model is developed for flood plain study purposes. Both a finite difference grid and an irregular triangle element integrated finite difference formulation are presented. The governing flow equations are approximately solved as a diffusion model coupled to the equation of continuity. Application of the model to a hypothetical dam-break study indicates that the approach can be used to predict a two-dimensional dam-break flood plain over a broad, flat plain more accurately than a one-dimensional model, especially when the flow can break-out of the main channel and then return to the channel at other downstream reaches.

Key Words: Unsteady flow, dam-break analysis, two-dimensional hydrodynamic model

INTRODUCTION

The probable flooding damages which may occur due to a dam failure is of concern to many civil engineers and planners. Not only do such studies provide a source of information for insurance and flood control studies, but the actual planning process for the construction of a dam site can be modified by the results of such a predictive analysis.

Generally, such dam-break studies can be completed by either scaled hydraulic models or by use of computer simulation. The most cost effective approach is to approximately solve the governing flow equations of momentum and continuity by computer simulation, where several situations can be considered by a fraction of the cost of a scaled prototype model.

For water courses in which flows can be classified as one-dimensional, several models are presented and verified in the literature. One widely used one-dimensional model which is utilized in this study is the K-634 model version developed by Land[1,2].

However for flow conditions which are truly two-dimensional, such as occur when massive dam-break flows exceed the water course channel capacities and excess flows break out and travel away from the water course, then a one-dimensional approximation may be inappropriate.

In this paper, a simple two-dimensional dam-break model is developed and applied to a hypothetical dam failure situation where channel break out flows are a major factor in the determination of a flood plain. The model is based on a diffusion approach where gravity, friction, and pressure forces are assumed to dominate the flow equations. Such an approach has been used earlier by Xanthopoulos and Koutitas[3] in the prediction of dam-break flood plains in Greece. In those studies, good results were also obtained in the use of the two-dimensional model in predicting one-dimensional flow quantities. The diffusion model has also been used to successfully model one-dimensional channel flows[4]. In another paper, Hromadka[5] considers a one-dimensional diffusion model and concluded that for low-to-moderate velocity flow regimes (i.e., less than approximately 25 feet/sec), the diffusion model is a reasonable approximation of the full dynamic wave formulation.

In this paper, two versions of the two-dimensional diffusion model are presented. A finite difference grid model is developed which equates each cell-centered node to a function of the four neighbouring cell nodal points. Additionally, the finite difference model is extended to an integrated finite difference analog based on irregular triangles using a nodal domain integration (NDI) control volume definition for each node. Both models are used to generate hypothetical flood plains, and these results are compared to a similar study based on the one-dimensional K-634 model.

A comparison and analysis of model results shows that the two-dimensional diffusion approach provides a more reasonable representation of two-dimensional flow effects than does a fully dynamic one-dimensional model, and that the diffusion model affords an easy-to-use predictive tool for the estimation of dam-break flooding depths over two-dimensional flood plains.

STUDY AREA DESCRIPTION

The Owens River drains the rugged eastern slopes of the Sierra Nevada Range, the Benton Range, and the western slopes of White Mountains located in Inyo County, California (Fig. 1). The head waters of the Owens River flows into Lake Crowley (formed by the Long Valley dam). The dam is an earthen type, 126 ft in height with a crest length of 595 ft, and a total storage capacity of 183,465 acre-feet of water. Downstream from Long Valley Dam, the Owens River is constrained in an incised canyon

Accepted June 1984. Discussion closes May 1985.

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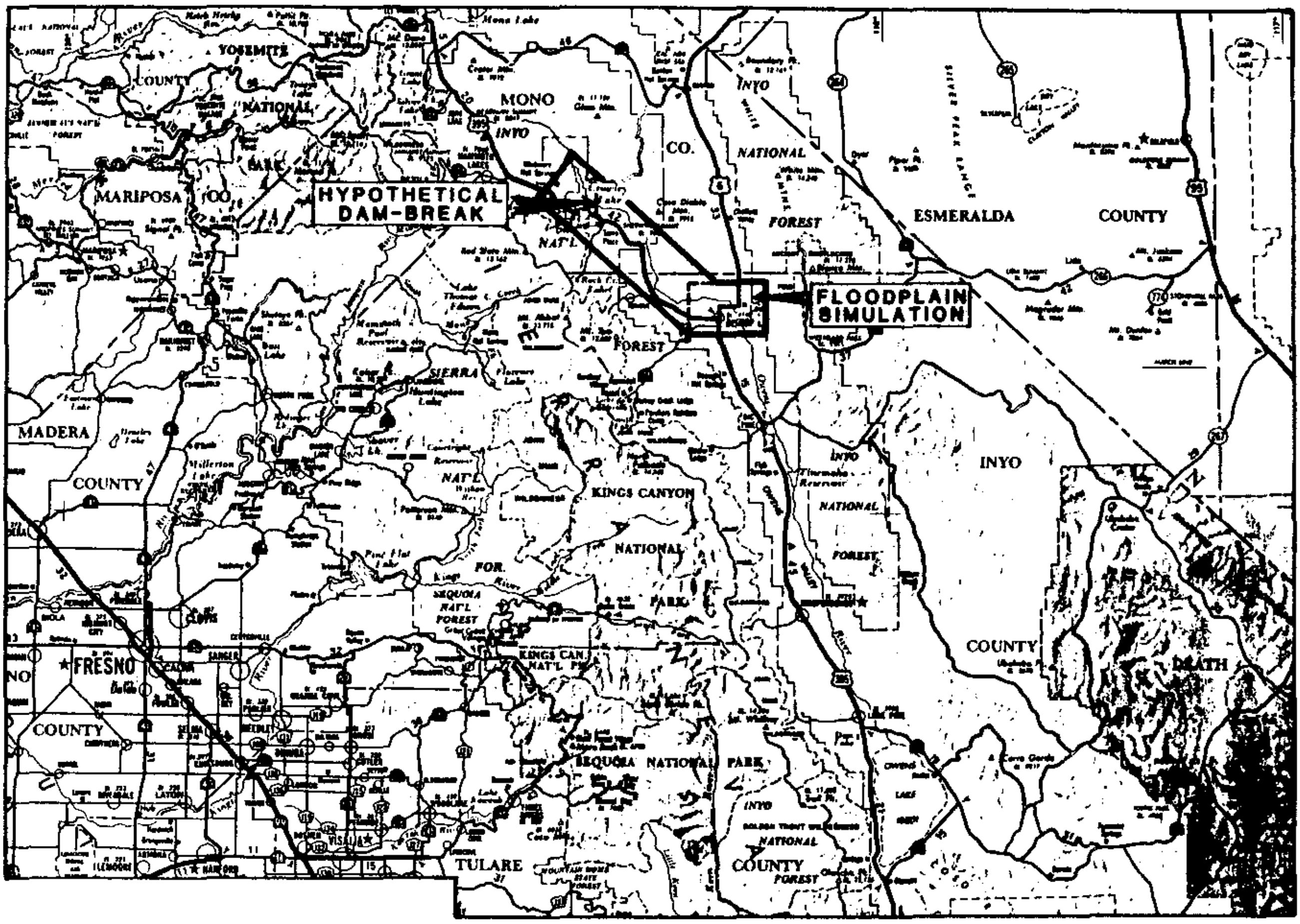

Fig. 1. Dam-break study location

of volcanic Bishop tuff formation. The river flows through the gorge to the canyon mouth, seventeen miles downstream from the dam. The river encounters several hydroelectric power plants in reaching the canyon mouth. However in this study, the effects of these obstructions in a dam-break analysis is assumed minor due to the relative magnitude of the flood volume compared to the volume afforded by each power plant.

Through the first several miles downstream from the canyon mouth, the river meanders through alluvial fans with a slope of 50 ft per mile. Four miles downstream from the canyon mouth, the river enters Owens Valley which is a flat alluvial valley with a slope of only 10 ft per mile. (The basin elevations range from 4300 ft to 3800 ft). The river curves around the City of Bishop about 17 miles downstream from the canyon mouth, and then meanders through Owens Valley for another 40 miles until it reaches Tinemaka Reservoir. Figures 1 and 2 illustrate the study location and vicinity.

One-dimensional analysis approach

In this section, the one-dimensional dynamic wave, implicit dam-break model K-634 developed by Land[1,2] is used to investigate a hypothetical dam failure and resulting flooding along the Owens River.

The K-634 model routes a flood hydrograph through a reservoir with a water level initially below the position where a breach in the retaining dam is assumed to occur. The routing of this hydrograph is accomplished by one of two procedures; namely, a hydrologic method (storage-continuity), or a hydraulic method such as discussed in Land[1,2]. When the water level in the reservoir reaches a preselected level, a breach in the dam is assumed to begin. The breach begins with a zero width, and widens and deepens during a specified time of failure. At the end of the failure, the breach is fully developed and is assumed to remain constant for the remainder of the simulation. The breach flow rates are computed based on a trapezoidal-shaped critical flow area. The outflow hydrograph at the dam is computed as the sum of the initial flow through dam structures and the flow through the breach.

The dam-break flood is then mathematically routed downstream by solving the one-dimensional Saint Venant flow equations using a nonlinear implicit finite-difference algorithm.

One-dimensional model data requirements

The K-634 data requirements fall into three categories. (1) In the reservoir: surface area versus elevation tables, or channel geometry and roughness; (2) at the dam: breach shape, duration of breach development, dam outlet structures stage-outflow ratings, and water-surface elevation when failure begins to occur; and (3) in the stream: channel geometry, roughness and state of flow (e.g. subcritical of supercritical). The upstream boundary condition for routing the flood downstream is the dam-break outflow hydrograph. The downstream boundary

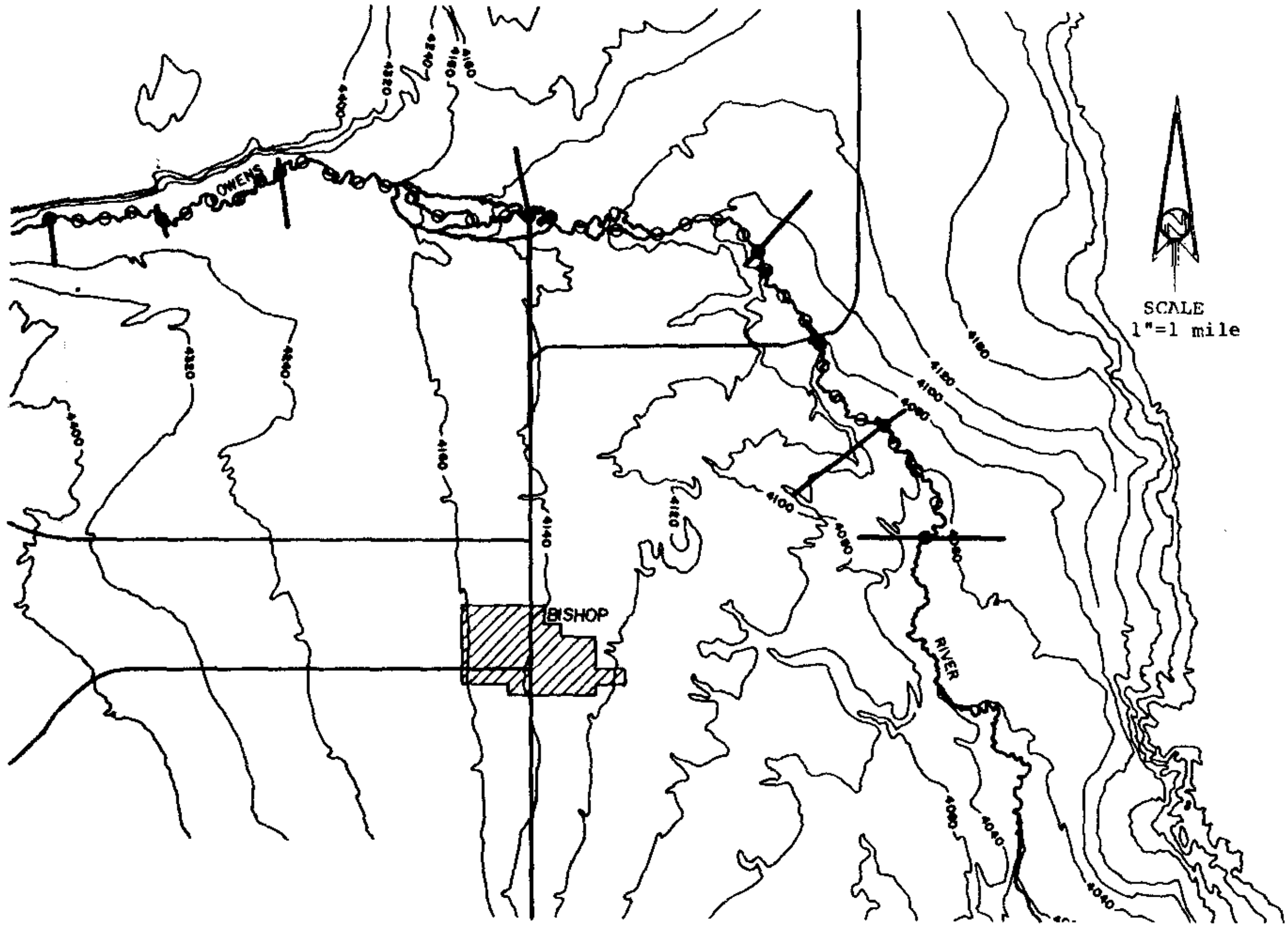

Fig. 2. Surveyed cross section locations on Owens River for use in K-634 model

condition is a dynamic stage-discharge relation which is computed from the full dynamic flow equations.

The one-dimensional model grid schemetization is shown in Fig. 2. The channel geometry of the reach of river was defined by field-surveyed cross sections where, for each section, a maximum of eight elevation versus top width data pairs were specified. Due to the relatively flat topography perpendicular to the main channel (see Figs. 2 and 6), the upper most portions of the channel cross sections were assumed to have a mild gradient. This modeling assumption results in a false confinement of channel flows due to a fictitious boundary being assumed on both sides of the channel. Consequently, predicted flood plains will be approximate at best, and requires additional interpretation. The effects of this modeling assumption will be further discussed in a subsequent section.

Mathematical formulation for two-dimensional model

The Saint Venant equations for two-dimensional flow may be written as

$$\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial H}{\partial t}=0 \tag{1}$$

$$\frac{\partial Q_x}{\partial t}+\frac{\partial}{\partial x}Q_x^2/A_x+\frac{\partial}{\partial y}Q_xQ_y/A_y+gA_x\left(S_{fx}+\frac{\partial H}{\partial x}\right)=0 \tag{2}$$

$$\frac{\partial Q_y}{\partial t}+\frac{\partial}{\partial y}Q_y^2/A_y+\frac{\partial}{\partial x}Q_yQ_x/A_x+gA_y\left(S_{fy}+\frac{\partial H}{\partial y}\right)=0 \tag{3}$$

where Q_x,Q_y are flow rates in x,y-directions; q_x,q_y, flow rates/length in x,y-directions; A_x,A_y, directional cross-section areas; S_{fx},S_{fy}, directional friction slopes; x,y,t, spatial and temporal coordinates; g, gravity; H, water surface elevation; where, for a grid discretization (Fig. 3), Q_x equals the product of q_x and the grid width, δ. The equation of continuity (equation (1)) is based on the assumption of constant fluid density with zero sources or sinks in the flow field. The x- and y-direction momentum relations (equations (2) and (3)) assume hydrostatic pressure distributions.

The local and convective acceleration momentum terms can be grouped together such that equations (1), (2) and (3) are rewritten as

$$m_z+\left(S_{fz}+\frac{\partial H}{\partial z}\right)=0, \qquad z=x,y \tag{4}$$

where m_z represents the sum of the first three terms in equations (2) and (3), and divided by gA_z. Assuming the friction slope to be approximated by steady flow conditions, the Manning's equation in inch-pound units can be used to estimate

$$Q_z=\frac{1.486}{n}A_zR_z^{2/3}S_{fz}^{1/2} \tag{5}$$

where A_x,A_y are directional flow areas; R_x,R_y, directional hydraulic radii. Equation (5) can be rewritten as

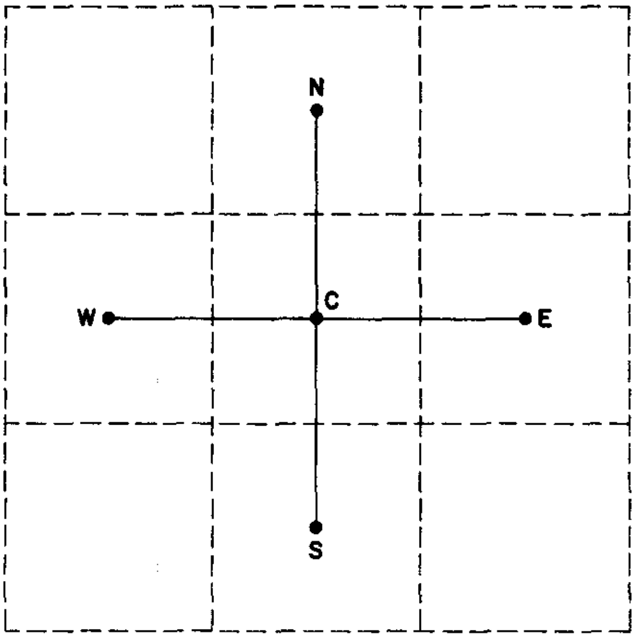

Fig. 3. Grid element nodal molecule

$$Q_z = -K_z \frac{\partial H}{\partial z} - K_z m_z, \qquad z = x,y \tag{6}$$

where

$$K_z = \frac{1.486}{n} A_z R_z^{2/3} \Big/ \left| \frac{\partial H}{\partial z} + m_z \right|^{1/2}, \qquad z = x,y \tag{7}$$

Hromadka[5], Akan and Yen[4], and Xanthopousos and Koutitas[3] assume m_x and m_y are both negligible, resulting in the simple diffusion model

$$Q_z = -K_z \frac{\partial H}{\partial z} \tag{8}$$

In equation (6), m_z can be retained in either the full form, or can include either the local of the convective acceleration component. However, Hromadka[5] showed that for one-dimensional dam-break flood waves on flat plains where flow velocities are generally below about 25 ft/sec m_z is generally small, and the loss of accuracy incurred by setting $m_z = 0$ may be considered acceptable when considering the modeling errors due to the wide range of model parameter variability which occurs in this type of analysis as a result of the assumed dam-break mode of failure, failure rate and shape, watershed friction factor distribution and variation during the dam-break, and other factors. Nevertheless, m_z can be included in equation (6) resulting in the complete Saint Venant formulation; however, the resulting computer computation effort increases by approximately 50% due to the cross-product evaluation of the convective acceleration terms and the local acceleration term approximations.

The proposed two-dimensional dam-break model is formulated by substituting equation (8) into the continuity equation giving

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} = \frac{\partial H}{\partial t} \tag{9}$$

If the momentum term groupings were retained, equation (9) would be written as

$$\frac{\partial}{\partial x} K_x \frac{\partial H}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial H}{\partial y} + S = \frac{\partial H}{\partial t} A \tag{10}$$

where A is the nodal control volume area and

$$S = \frac{\partial}{\partial x}(K_x m_x) + \frac{\partial}{\partial y}(K_y m_y)$$

and K_x, K_y are also functions of m_x, m_y respectively.

Numerical model formulation (grid elements)

For uniform grid elements, the numerical modeling approach used is the integrated finite difference version of the nodal domain integration (NDI) method. For grid elements, the NDI nodal equation is based on the usual nodal system shown in Fig. 3. Flow rates along the boundary Γ are estimated using a linear trial function assumption between nodal points. For a square grid of width δ

$$Q|_{\Gamma_E} = -(K_x|_{\Gamma_E})(H_E - H_C)/\delta \tag{11}$$

where

$$K_x|_{\Gamma_E} = \begin{cases} 1.486(AR^{2/3}/n)_{\Gamma_E} / |(H_E - H_C)/\delta|^{1/2}; \ \bar{D} > 0 \\ 0; \ \bar{D} \leqslant 0 \text{ or } |H_E - H_C| < 10^{-3} \end{cases} \tag{12}$$

In equation (12), the terms A, R, and n are evaluated at the average flow depth defined by $\bar{D} = (D_E + D_C)/2$, and $n = (n_E + n_C)/2$. Additionally, the denominator of K_x is checked such that K_x is set to zero if $|H_E - H_C|$ is less than a tolerance such as 10^{-3} ft.

The model advances in time by an explicit approach

$$\mathbf{H}^{i+1} = \mathbf{K}^i \mathbf{H}^i \tag{13}$$

where the assumed dam-break flows are added to the specified input nodes at each timestep. After each timestep, the conduction parameters of equation (12) are re-evaluated, and the solution of equation (13) reinitiated. Using grid sizes with uniform lengths of one-half mile, timesteps of size 3.6 sec were found satisfactory.

Numerical model formulation (irregular triangle elements)

The simplex triangle element can be subdivided into nodal domains such as shown in Fig. 4. The total assemblage of triangle elements results in NDI control volumes for each nodal point such as shown in Fig. 4. Assuming a linear trial function on each simplex element, the x- and y-direction flow terms are computed by

$$Q_x = -(K_x|_*)\left(\frac{\partial H}{\partial x}\right)(|\bar{\Gamma}_y|) \tag{14}$$

where K_x is evaluated similar to equation (12) at the triangle centroid (*), and $|\bar{\Gamma}_y|$ is the length of the NDI boundary projected on the y-axis (see Fig. 5). Hromadka and Guymon[6] show that the assemblage of NDI flow rate terms result in an element conduction matrix system $\mathbf{K}_x^e\mathbf{H}^e + \mathbf{K}_y^e\mathbf{H}^e$ which is identical to the well known Galerkin conduction matrix for the simplex element. Integrating the linear trial function on each nodal domain

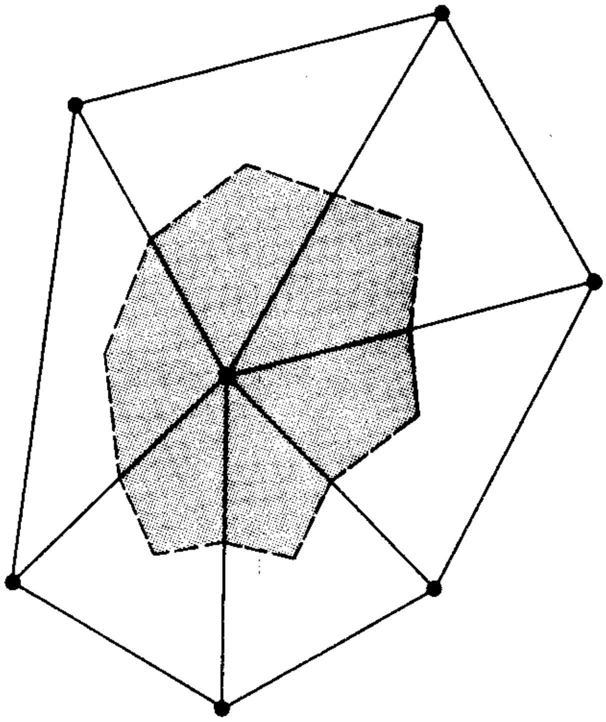

Fig. 4. NDI control volumes

results in the mass matrix

$$C^e = \frac{A^e}{3(\eta+2)} \begin{bmatrix} \eta & 1 & 1 \\ 1 & \eta & 1 \\ 1 & 1 & \eta \end{bmatrix} \mathbf{H}^e \qquad (15)$$

where the superscript (e) refers to simplex element (e); A^e is the element area; and η is a mass lumping factor. Using $\eta = (2, 22/7, \infty)$ results in a Galerkin, subdomain integration, and an integrated finite difference algorithm, respectively.

In the applied model, $\eta = \infty$ was used with the explicit time advancement of equation (13). At each timestep, K_x and K_y are evaluated at each triangle element's centroid, and flux terms are computed from equation (14). The change in the nodal value of H is then estimated directly from the timestep duration of net inflow into the NDI control volume.

As stated before, the total momentum term groupings of (m_x, m_y) can be retained in the diffusion model, using an iteration approach in the explicit model until an acceptable balance of values is achieved between timesteps.

Two-dimensional model data requirements

Both of the two-dimensional models require only the natural ground topography and estimated Manning's friction factor for data input. Consequently, the diffusion model will provide useful dam-break flood-plain estimates with only minimum data requirements which are usually readily available. For the grid model, a nodal molecule associates the central node to the north, east, south, and west nodal points. Zero flow boundary conditions are easily accommodated by entering a zero for the appropriate nodal molecule position. The nodal point elevation is based on an estimated average elevation for the assumed grid cell.

The NDI triangular element model requires nodal point coordinates and elevations, and averaged triangular element friction factors.

Both models include a critical depth (specified) boundary condition whereby flow depths may be assumed to correspond to critical flow conditions.

Finally, the inflow hydrograph may be specified at one or more nodal points. In this study, the form of this hydrograph was specified as the change in flow rate per 5-minute unit period. This time derivative of inflow is used to provide a smooth inflow hydrograph stepped according to the specified model time step.

CASE-STUDY RESULTS AND DISCUSSION

The K-634 model was initially applied to the steep canyon reach immediately downstream of the assumed dam-break. The modeling results indicated negligible attenuation of the flood wave peak and, consequently, subsequent studies of the two-dimensional plain were assumed to have the dam-break located at the downstream point of the canyon, neglecting the effects of the long canyon reach.

Applying the K-634 model to computing the two-dimensional flow was attempted by means of the one-dimensional nodal spacing shown in Fig. 2. Cross sections were obtained by field survey, and the elevation data used to construct nodal point flow-width versus stage diagrams for use in the K-634 model. A constant friction factor (Manning's) of 0.04 was assumed for study purposes. The assumed dam-break failure reached a peak flow rate of 420 000 cfs within one hour, and returned to zero flow 9.67 hours later. The resulting K-634 flood plain limits is shown in Fig. 6. As discussed in a previous section, a slight gradient was assumed for the topography perpendicular to the main channel. The motivation for specifying such a gradient was to limit the channel floodway section in order to approximately conserve the one-dimensional momentum equations. As a result of this assumption, fictitious channel sides are included in the K-634 model

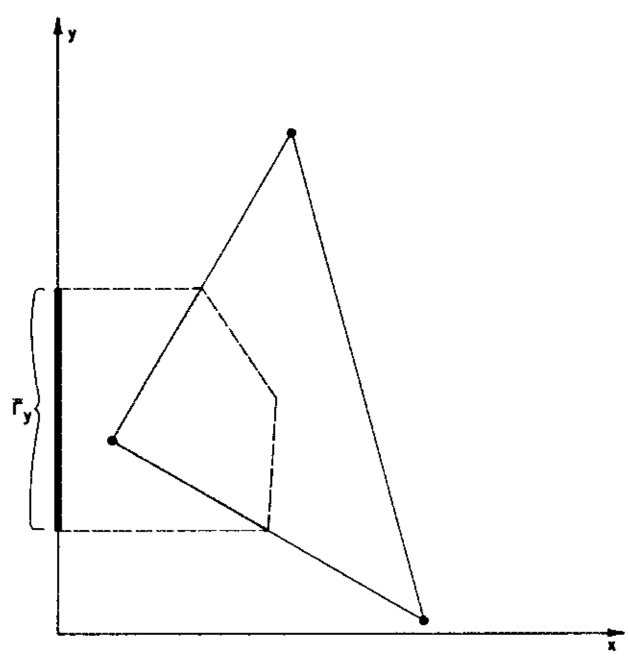

Fig. 5. Projection of nodal domain onto y-axis

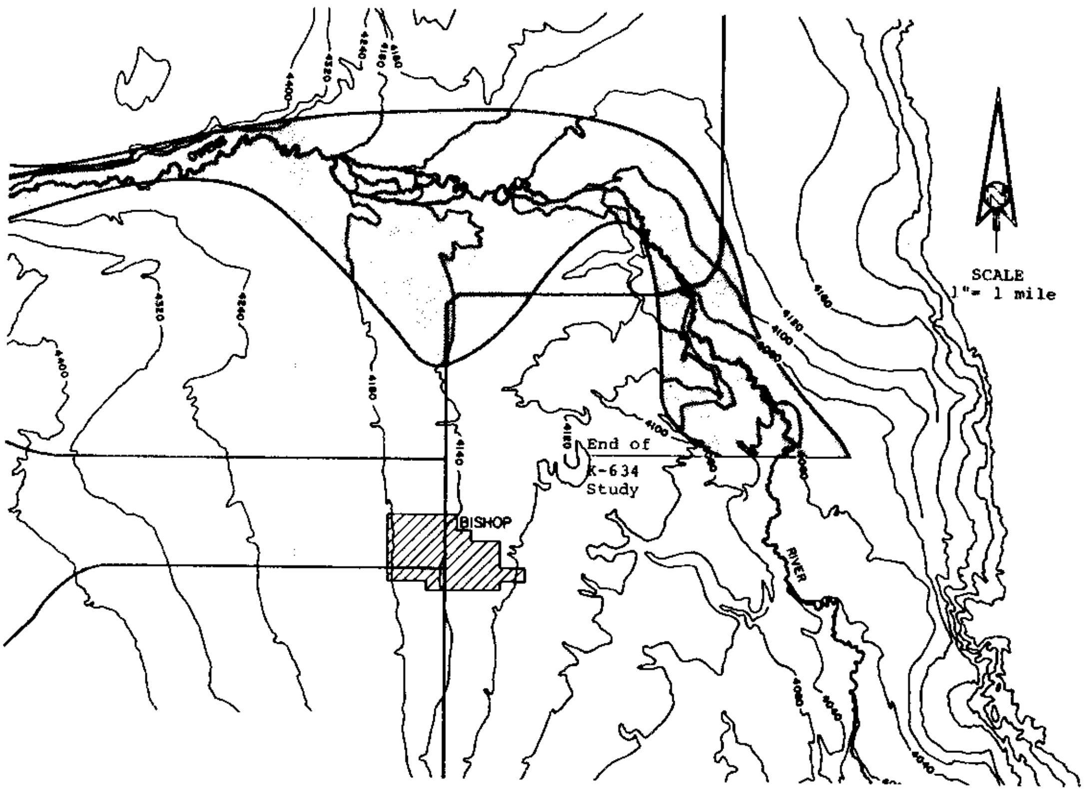

Fig. 6. Floodplain computed from K-634 model

study which results in an artificial confinement of the flows. Thus a narrower flood plain is delineated (such as shown in Fig. 6) where the flood flows are falsely retained within a hypothetical channel confine. An examination of the flood depths given in Fig. 9 indicates that at the widest flood plain expanse of Fig. 6, the flood depth is about 6-feet, yet the flood plain is not delineated to expand southerly, but is modeled to terminate based on the assumed gradient of the topography towards the channel. Such complications in accommodating an expanding flood plain when using a one-dimensional model are obviously avoided by using a two-dimensional approach.

The two-dimensional diffusion models were applied to the total flood plain area using the grid and triangular discretizations shown in Figs. 7 and 8, respectively. The same inflow hydrograph produced by the K-634 model was used for both two-dimensional simulations. Again, the Manning's friction factor of 0.04 was used. The resulting flood plain is shown in Fig. 7 for the $\frac{1}{4}$-square-mile grid model. The 168 triangular element NDI model resulted in a similar predicted flood plain (Fig. 8).

Comparisons of predicted maximum water elevations are shown in Fig. 9 which plots K-634 modeling results and the two-dimensional modeling results. The two approaches are comparable except at points shown as *A* and *B* in the figure. Point *A* corresponds to the predicted breakout of flows away from the Owens River channel with flows traveling southerly towards the City of Bishop. As discussed previously, the K-634 predicted flood depth corresponds to a flow depth of 6 feet (above natural ground) which is actually unconfined by the channel. The natural topography will not support such a flood depth and, consequently, there should be southerly breakout flows such as predicted by the two-dimensional models. With such breakout flows included, it is reasonable that the two-dimensional models would predict a lower flow depth at point *A*.

At point *B*, the K-634 model predicts a flood depth of approximately 2 feet less than the two-dimensional models. However at this location, the K-634 modeling results are based on cross-sections which traverse a 90-degree bend. In this case the K-634 model will over-estimate the true channel storage, resulting in an under-estimation of flow depths.

In comparing the various model predicted flood depths and delineated plains, it is seen that the two-dimensional diffusion model produced more reasonable predictions of the two-dimensional flood plain characteristics which are associated with broad, flat plains such as found at the study site than the one-dimensional model. The two-dimensional model affords approximation of channel bends, channel expansions and contractions, flow breakouts, and the general area of inundation. Additionally, the two-dimensional modeling approach allows for the inclusion of return flows (to the main channel) which result due to upstream channel breakout flows, and other two-dimensional flow effects without the need for special modeling accommodations which would be required when using a one-dimensional model.

Model sensitivity

The sensitivity of the overall modeling approach is illustrated for the case of K-634 by examining the three parameters of nodal spacing, Manning's *n*, and dam-break time of failure. The base run values are a nodal spacing of 0.25 mile, Manning's friction factor of 0.04, and a 1-hour dam-failure duration.

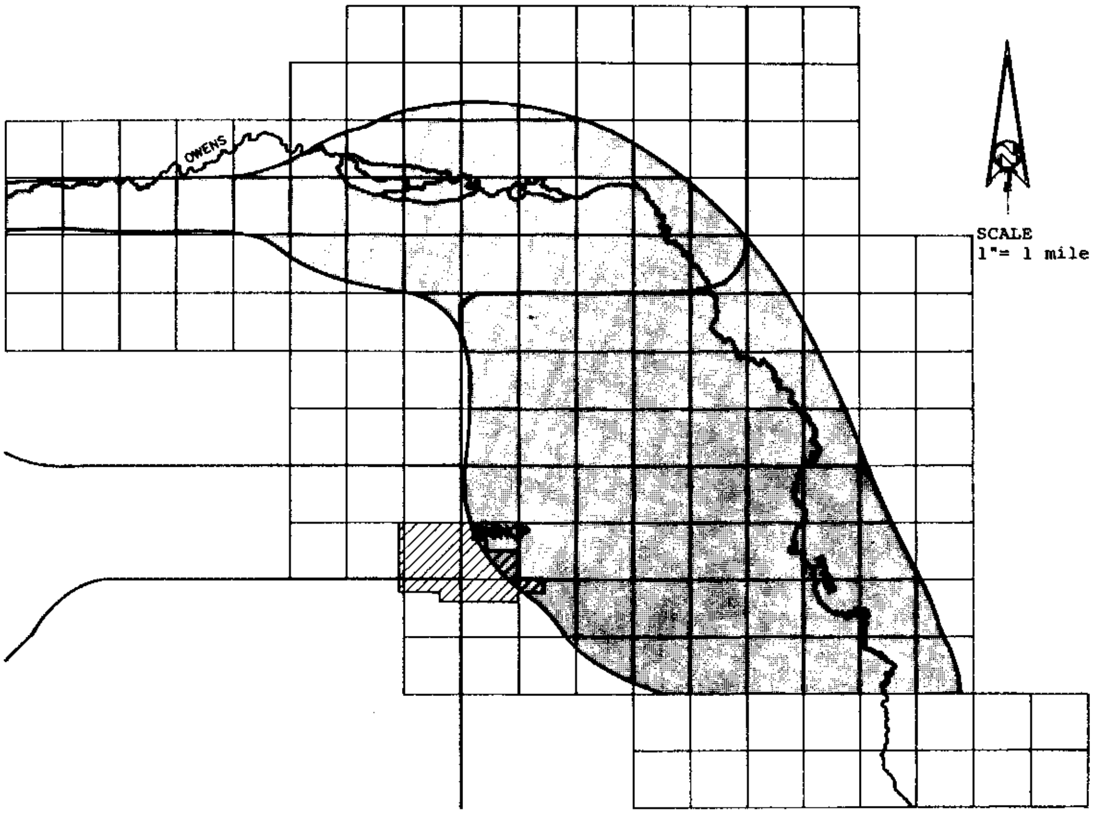

Fig. 7. Floodplain and grid layout for two-dimensional diffusion model

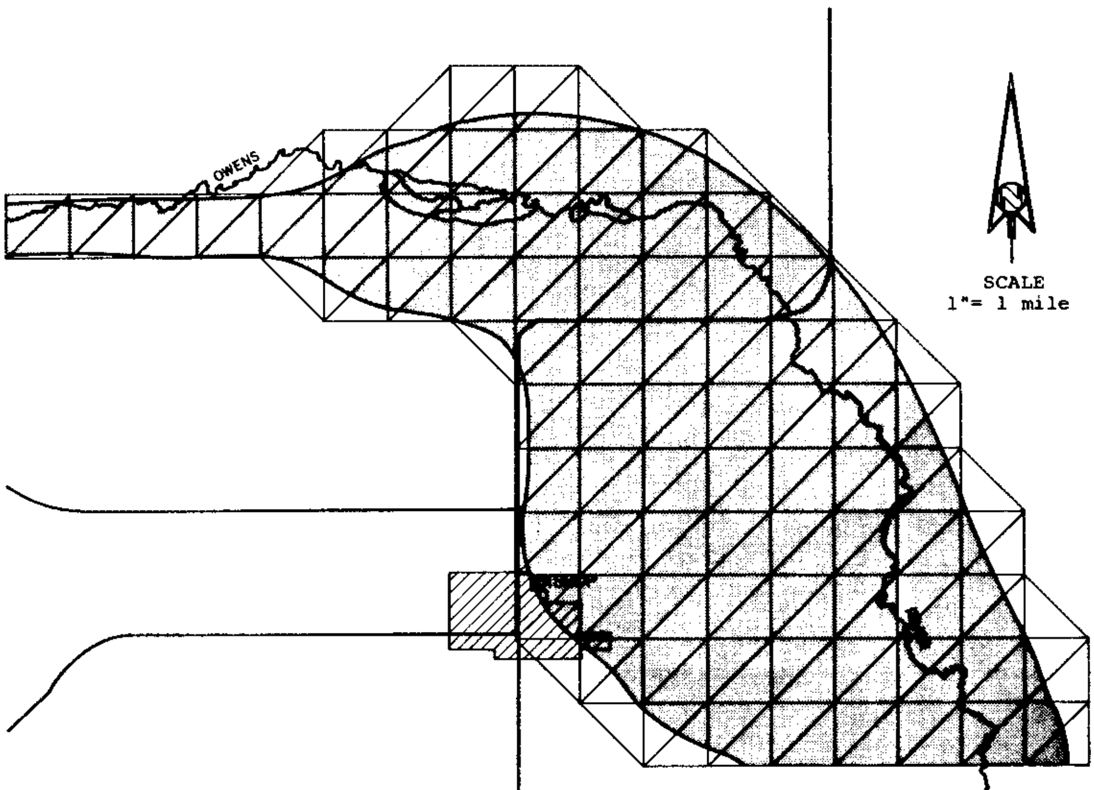

Fig. 8. Floodplain and triangle layout for two-dimensional diffusion model (108 nodes)

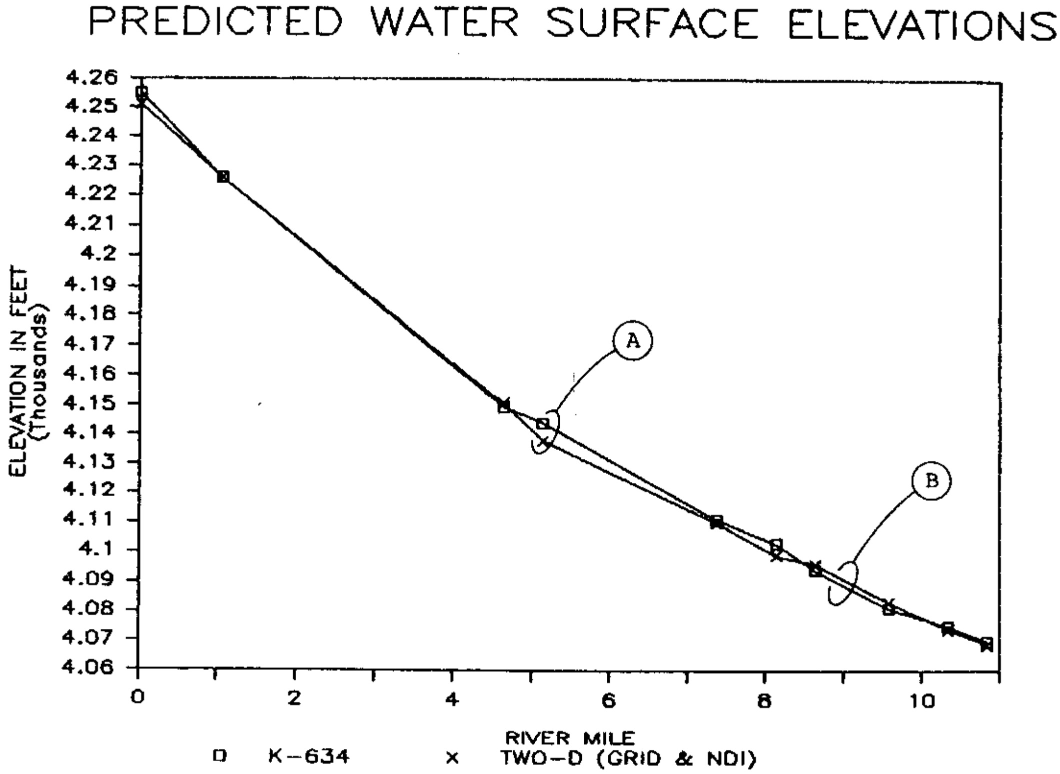

Fig. 9. Comparison of modeled water surface elevations

Table 1. K-634 sensitivity analysis

	River Mile			
	2.0		8.4	
	Maximum flowrate (cfs)	Maximum elevation	Maximum flowrate (cfs)	Maximum elevation
Node spacing				
0.25 mi*	408 038	4208.80	384 717	4098.58
0.30 mi	412 266	4208.98	386 164	4098.60
0.40 mi	412 082	4209.11	385 878	4098.59
0.50 mi	412 129	4209.12	384 828	4098.99
Time of failure				
0.50 hr	420 301	4209.04	388 965	4098.64
1.0 hr*	408 038	4208.80	384 717	4098.58
1.5 hr	400 352	4208.66	380 050	4098.49
1.75 hr	399 380	4208.64	377 584	4098.45
2.0 hr	394 846	4208.56	374 338	4098.39
Manning's n				
0.04*	408 038	4208.80	384 717	4098.58
0.045	411 221	4209.82	382 273	4099.35
0.05	410 386	4210.74	378 310	4100.05

* Base run used for flood plain study purposes

It can be seen in Table 1 that the most sensitive parameter is the friction factor, followed by the relatively insensitive parameters of time of failure and the nodal spacing. Similar sensitivity is found for the diffusion models.

CONCLUSIONS

A simple two-dimensional diffusion model was developed which can be based on either a finite-difference grid or a NDI irregular triangular element discretization. The keystone of the two-dimensional model is the diffusion version of the Saint Venant flow equations. Application of the model to the prediction of two-dimensional dam-break flood waves over broad, flat plains indicate that the diffusion model provides a significant advantage over the corresponding one-dimensional models in the study of flood plains such as occurs from a dam-break. The two-dimensional model is simple to use, requires readily available data, and does not need special modeling techniques to approximate two-dimensional flood flow effects.

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