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A ONE DIMENSIONAL FROST HEAVE MODEL BASED UPON SIMULATION OF SIMULTANEOUS HEAT AND WATER FLUX

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SUMMARY

A one-dimensional frost heave model is presented for unidirectional freezing in moist silts with a water table present. Frost heave is computed based upon a macro-thermodynamic model that simulates heat and moisture transfer from an element of soil undergoing freezing. It is assumed that a portion of water in the element will not freeze, and all water in addition to this amount that freezes in excess of the soil porosity results in a corresponding heave. The freezing process is assumed to be isothermal. Simultaneous heat and water flux are simulated by a Galerkin finite element analog of the heat equation, including convective component, and the water flux equation based upon total energy head as the state variable. The dynamic component of the problem is simulated by the Crank-Nicholson procedure, and non-linear parameters are estimated on the basis of element centered state variable values which are generated by the quadratic shape functions used. To avoid instability, the moisture sink term in the water flux equation that arises as a result of water freezing is eliminated and handled as a thermodynamic bookkeeping quantity.

INTRODUCTION

Numerical models of soil freezing and thawing may be conveniently divided into two groups. The first group concerns only thermal effects where moisture content is considered a static phenomena. Nakano and Brown (19) are an excellent example of this approach. They base their analysis of soil water freezing on the apparent heat capacity concept where the temporal thermal term is given by

$$\frac{\partial(C_m T)}{\partial t} = \left(C_m - L\frac{\rho_i}{\rho_w}\frac{\partial\theta_i}{\partial T}\right)\frac{\partial T}{\partial t} \qquad (1)$$

where C_m is the volumetric heat capacity of the soil-ice-water mixture, T is the temperature, t is the time, L is the latent heat of fusion, ρ_i and ρ_w are density of ice and water, θ_i is the volumetric ice content. The term in parenthesis is often referred to as apparent heat capacity, C_a. Generally, eq. 1 is integrated or averaged over a narrow freezing zone of finite thickness in order to accurately predict the temperature profile in a one-dimensional soil. In their study, Nakano and Brown noted that convected moisture may carry heat which would ultimately effect their numerical solution. Another problem is that if moisture is mobile, the computation of thermal parameters may be in error unless these parameters include the effects of a varying unfrozen water content. For instance, the thermal conductivity and volumetric heat capacity of a partly frozen soil may be approximated by the DeVries (5) relationship

$$K_T = \Sigma \gamma_j \theta_j K_j / \Sigma \gamma_j \theta_j \quad (2)$$

$$C_m = \Sigma \theta_j C_j / \Sigma \theta_j$$

where K_T is the thermal conductivity of the mixture, γ_j is a soil weighting factor, θ_j is the relative volumetric content of the jth constituent, and K_j is the thermal conductivity of the jth constituent, C_m is the volumetric heat capacity of the mixture, and C_j is the volumetric heat capacity of the jth constituent. Thus, neglecting a change in water content would introduce errors in the K_T or C_m estimation.

The second group of models to be considered simulate the simultaneous flux of fluid and heat in freezing and thawing soils and are the main concern of this paper. These one-dimensional models are based upon the concept that moisture flux in freezing and thawing soils can be analyzed by unsaturated flow theory. Either pore pressure or water content may be the state variable. For example, the pore pressure relation for water flux alone, Richard's Equation has been used; i.e.,

$$\frac{\partial}{\partial x}\left(K_H \frac{\partial \Psi}{\partial x}\right) - \frac{\partial}{\partial x}(K_H) = \frac{\partial \theta_u}{\partial t} + \frac{\rho_i \partial \theta_i}{\rho_w \partial t} \quad (3)$$

where x is the coordinate (positive downward), Ψ is the pore pressure head, K_H is the Darcy hydraulic conductivity, θ_u and θ_i are the volumetric unfrozen water and ice content respectively, ρ_i and ρ_w are the ice and water density, and t is time. Coupled one-dimensional heat transport is modeled by

$$\frac{\partial}{\partial x}\left(K_T \frac{\partial T}{\partial x}\right) - C_W v \frac{\partial T}{\partial x} = C_m \frac{\partial T}{\partial t} - L \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t}$$

$$= C_a \frac{\partial T}{\partial t} \quad (4)$$

Harlan (13) presented a one-dimensional model of heat and moisture transport using eqs. 3 and 4, apparent heat capacity, and various ancilliary relationships to define fluid and heat parameters. His model was numerically solved by a finite difference analog. Guymon and Luthin (12) developed a similar model to that of Harlan using eqs. 3 and 4, however, they solved the heat and moisture flux equations with a finite element analog. Both methods were similar and for the first time included the gravitational component of fluid flux for a vertical column and also included the convective heat moisture transport component that was absent from the earlier models such as Nakano and Brown (19). More recently, the application of heat and fluid transport models have been investigated for predicting frost heave. Dempsey (3) presented a one-dimensional model of heat and moisture flux, similar to that of Harlan, and Guymon and Luthin but he also included temperature driven moisture flux in the liquid and vapor phases. His model, solved by a finite difference approximation, was developed to also study the frost heave phenomena. Sheppard, et al (21) also used eqs. 3 and 4 and the apparent heat capacity concept to model frost heave; their model is solved by a finite difference approximation. Heave is assumed to occur when the sum of ice and water content exceed the soil porosity. Overburden effects are modeled by the Groenevelt and Kay (9) approximation. Taylor and Luthin (23) use eqs. 3 and 4 but delete the gravitational component from eq. 3 and omit the convective component from eq. 4 to approximate frost heave. They use a finite difference approach and use the apparent heat capacity concept. Heave is allowed to occur when ice content exceeds 85 percent of the soil porosity. Various ancilliary functions are used to estimate parameters. They closely approximate data obtained by Jame and Norum (17) for a horizontal soil column. Jame (16) also applied what is essentially Harlan's model to his data with good results after an apparent instability problem in the early computations. Both Taylor and Luthin, and Jame assume convective heat transport as negligible, which apparently is reasonably accurate for the relatively short (72 hours or less) time periods considered. Dudek and Holden (7) use equations similar to eqs. 3 and 4, but neglect heat convection, to solve the heat and moisture problem. They use an expanding-contracting finite difference mesh to solve the equations. Latent heat effects are modeled by the apparent heat capacity method and the so-called capillary theory of frost heave is used. They closely approximate laboratory data for a short time span, less than 80 hours.

Kinosita (18) and Outcalt (20) have considered models of frost heave in soils. Kinosita's work primarily considered *in situ* measurements of soil temperature and water table drop to relate heat and moisture flux to heave and heave pressure as measured in large outdoor tanks containing silt and water. Outcalt considers the thermodynamics of ice segregation with ancilliary heat transport computations. Outcalt (20) applies a simple energy balance to predict ice segregation.

A second approach is to lump the latent heat effects into an isothermal process. Guymon and Berg (10) and Guymon and Hromadka (11) use equations similar to eqs. 3 and 4 to model heat and water transport but handle latent heat effects as an isothermal process in which an accumulation heat budget is used in the computational algorithm. A Galerkin finite element analog is used to solve the equations of state, and linear and quadratic shape functions are used.

A third category of the modeling effort is by Hwang (15) who has developed a quasi-steady state frost model for application to chilled pipelines. He assumes a pore water distribution near the freezing front in order to estimate water flux. Heat transport includes both moisture flux and latent heat effects. Heave is modeled by a consideration of a stress-strain relationships for frozen and unfrozen soil, and consolidation theory issued to adjust heave calculations. He assumes that heave has an upper bound and that heave will not occur once the freezing front propogation becomes nil. This concept is somewhat contrary to generally accepted theory which was recently reviewed by Takagi (22).

MODEL BASIS

As discussed above, most investigators use eqs. 3 and 4 to model the simultaneous movement of heat and water in a freezing soil column. However, when modeling of these equations involve numerical problems, the equations are specialized to eliminate the convective components; e.g. a horizontal column is considered and it is assumed convected heat is negligible (Taylor and Luthin (23) and Jame (16) are examples of this approach). In both of these cases, and also in the case of Sheppard et al (21) a single valued relationship between unfrozen water content and temperature is assumed (e.g. Anderson et al (1)). It is relatively easy to show that eqs. 3 and 4 can be combined so that only one equation need be solved if the apparent heat capacity approach is used. Numerical models employing this approach require exceedingly small time steps, on the order of seconds, and small descretization, on the order of centimeters. Instability problems may result for lengthy simulations, involving time spans of a week or more. The computational difficulty is that latent heat effects in freezing and thawing soil often times overshadow the basic parabolic nature of eq. 3 when latent heat is directly incorporated into a numerical analog specifically designed for a parabolic equation. On the basis of these problems and the fact that we desire to produce stable, accurate solutions over time spans of a year or more, we have concluded that the isothermal approach to modeling latent heat effects is superior. Additionally, there are ancilliary benefits to the isothermal approach; e.g. the computation of moisture states is enhanced and made highly stable.

Consider the heat budget ΔQ required to alter a unit volume soil-water-ice mixture by a temperature change of dT in a time interval of dt,

$$\Delta Q = C_m \, dT - L \frac{\rho_i}{\rho_w} \frac{\partial \theta_I}{\partial t} \, dt \qquad (5)$$

Equation 5 can be rewritten as

$$\Delta Q = C_m \frac{\partial T}{\partial t} \, dt - L \frac{\rho_i}{\rho_w} \frac{\partial \theta_I}{\partial t} \, dt \qquad (6)$$

where temperature is a differentiable function of time.

Application of eq. 6 to problems where the ice content of the soil can be assumed to be a differentiable function of temperature permits the rewriting

$$\Delta Q = \left(C_m - L \frac{\rho_i}{\rho_w} \frac{\partial \theta_I}{\partial T}\right) \frac{\partial T}{\partial t} \, dt \qquad (7)$$

which establishes the apparent specific heat formulation of eq. 1.

Inclusion of the moisture convection term in the heat transfer equation, however, necessitates consideration of the moisture flux on the term, $\partial \theta_I / \partial T$. The formation of the ice lenses in a freezing soil, where the temperature variations are small with respect to time, indicate that changes in ice content are not necessarily dependent on temperature changes. The processes of moisture movement within the freezing soil mass (Dirksen and Miller (6)), and the formation of ice lenses illustrate the capability of a soil mixture to be near a state of thermal equilibrium wherein the heat conduction loss rate is approximately equal to the rate of heat evolution from converting the moisture influx into ice. Hence, the soil mixture would be described by a static temperature model where

$$\frac{\partial \theta_I}{\partial t} \neq \frac{\partial \theta_I}{\partial T} \frac{\partial T}{\partial t} \qquad (8)$$

The apparent specific heat capacity would be undefined in an approximately static thermal regime, and the right side of eq. 4 would not be valid.

Assuming that the convected moisture enters the soil system at approximately the temperature of the freezing point depression, a small drop in temperature ΔT (in time Δt) of the soil-ice-water mixture indicates a heat budget of

$$\Delta Q = L[\theta' - \theta(T_o)] - L \frac{\partial \theta_I}{\partial T} \Delta T - C_m \Delta T \qquad (9)$$

where θ' is the volumetric moisture content at the beginning of the process; $\theta(T_o)$ is the volumetric water content described by the soil freezing curve for temperature T_o, T_o is the initial temperature of the system, ΔT is the temperature drop (assumed negative), and ΔQ is the heat lost from the system during the step Δt.

From eq. 9, three cases arise from a strictly freezing process

$$L\,[\theta' - \theta(T_o)] = \Delta Q \tag{10a}$$

$$L\,[\theta' - \theta(T_o)] > \Delta Q \tag{10b}$$

$$L\,[\theta' - \theta(T_o)] < \Delta Q \tag{10c}$$

Equation 10a is associated with isothermal freezing in a static thermal regime. Equation 10b also indicates isothermal freezing with no temperature variation but additionally indicates possible moisture accumulation. Equation 10c occurs with a temperature change of the system ΔT (during time step Δt) determined frem eq. 9 as

$$\Delta T = \frac{L[\theta' - \theta(T_o)] - \Delta Q}{L\frac{\partial \theta_I}{\partial T} + C_m} \tag{11}$$

The ice accumulation term is calculated by

$$\frac{\rho_i}{\rho_w}\frac{\partial \theta_I}{\partial t}\,dt = \begin{cases} \frac{\Delta Q}{L}, & \Delta Q \leq L\,[\theta' - \theta(T_o)] \\ [\theta' - \theta(T_o)] - \frac{\partial \theta}{\partial T}\Delta T, & \Delta Q > L[\theta' - \theta(T_o)] \end{cases} \tag{12}$$

Jame (16) approximated $\theta(T)$ by a set of linear functions defined on a discretized thermal domain. Assume for an appropriate temperature subdomain for $T_a \leq T \leq T_b$ that moisture content is given by

$$\theta(T) = \beta_o + \beta_1 T \tag{13}$$

where β_o, β_1 are constant soil parameters, then

$$\frac{\partial \theta(T)}{\partial T} = \beta_1 \tag{14}$$

Thus, for a small change in temperature ΔT (ΔT negative), eqs. 9 and 14 may be combined as

$$\Delta Q = \{L[\theta' - \theta(T_o)] - L\beta_1 \Delta T\} - C_m \Delta T \tag{15}$$

where the term in brackets represents an isothermal freezing process.

The terms within the parenthesis of eq. 15 may be approximated numerically by decoupling the ice formation term from the general heat transfer equation (Guymon and and Hromadka (11)) and allocating the subsequent heat evolution to a latent heat budget matrix. As the ice content increases, the thermal and moisture parameters are adjusted. Ice formation is interpreted as a moisture sink in the moisture transfer relation. Only when the necessary heat evolution has occurred, is the soil-water mixture's temperature allowed to recede below the freezing point depression, hence modeling the isothermal phase change process.

The above described isothermal approach together with eq. 4 are used in our model except C_a in eq. 4 is replaced by, C_m, the volumetric heat capacity of the mixture of soil, ice, and water. The thermal conductivity, K_T, is estimated by eq. 2 and velocity flux is computed by Darcy's law; i.e.,

$$v = -K_H \frac{\partial \varphi}{\partial x} \tag{16}$$

Auxiliary conditions of the form

$$\begin{aligned} T\,(x=0,\ t) &= T_u \\ T\,(x=L,\ t) &= T_L \\ T\,(x,\ t=0) &= T_o \end{aligned} \tag{17}$$

are employed. All parameters are assumed to be single valued. Effects of dissolved salts on the unfrozen water and freezing point depression are not incorporated in the model.

Although Guymon and Luthin (12) and Guymon and Berg (10) successfully used eq. 3 to model moisture movement in a vertical freezing column process, subsequent application of eq. 3 to situations where a water table is present or near steady state disclosed errors using a finite element approach. The problem relates to the necessity of linearizing eq. 3 in the finite element formulation. This problem when coupled with the use of the Crank-Nicholson time solution procedure resulted in the erroneous calculation of moisture flux due to gravity. Another problem with finite element formulations of equations like eq. 3 is that the gravity component, the so-called nonsymmetrical or convective component, results in nonsymmetrical matrices, increasing storage requirements. While this is not a great problem for one-dimensional solutions, it is for multi-dimensional problems, which are our ultimate objective to deal with.

Dudek and Holden (7) seem to have advocated the use of the energy head equation; however, to avoid its solution, they assume a saturated soil column below the freezing front and negligible fluid flux in the frozen zone. The equation of water flux in an unsaturated media is easily derived by considering continuity and assuming Darcy's law; i.e.

$$\frac{\partial[K\partial\phi/\partial x]}{\partial x} = \frac{\partial\theta_u}{\partial t} \tag{18}$$

where the variables have been previously defined. It is assumed that liquid driven by the hydraulic head gradient is the dominant fluid flux phenomena. Vapor driven by thermal and hydraulic gradients and liquid driven by thermal gradients are generally several orders of magnitude lower than water flux as defined by eq. 18. Following the procedure outlined by Guymon and Luthin, (12), θ_u is estimated by Gardner's (8) relationship

$$\theta_u = \theta_o/(A_w|\psi|^n + 1), \ \psi < 0 \tag{19}$$

and K_H may be estimated by

$$K_u = K_o/(A_K|\psi|^m + 1), \ \psi < 0 \tag{20}$$

where θ_o is the porosity, K_o is the saturated hydraulic conductivity, and A_w, A_K, n and m are constants for a particular soil assuming single valued functions for eqs. 19 and 20. The moisture sink term, representing the variation of water content due to phase change is not included because we have found that the solution of eq. 18 is highly unstable in a freezing soil since the sink term is so large. The water sink term is handled as a bookkeeping quantity, estimated from the latent heat budget computation; i.e.

$$\frac{\rho_i}{\rho_w}\frac{\partial\theta_i}{\partial t} = \frac{1}{L}\frac{\Delta Q}{\Delta t} \tag{21}$$

Equation 18 is rewritten using eq. 19 for computational convenience

$$\frac{\partial[K\partial\phi/\partial x]}{\partial x} = \theta^* \frac{\partial\phi}{\partial t} \tag{22}$$

where

$$\theta^* = \frac{\partial\theta_u}{\partial\psi}$$

and (23)

$$\frac{\partial\phi}{\partial t} = \frac{\partial\psi}{\partial t}$$

where $\phi = \psi - x$ (x positive downward).

Equation 22 is subject to the auxiliary conditions

$$\partial\phi/\partial x \ (x=0, t) = 0$$

$$\phi \ (x=L, t) = \phi_L \tag{24}$$

$$\phi \ (x, t=0) = \phi_o$$

NUMERICAL METHOD

The finite element method is uded to solve eqs. 3 and 22 subject to their respective auxiliary conditions. The finite element approach used is the Galerkin version of the weighted residual process, Desai (4). The solution domain is discretized into the union of n rigid finite elements (i.e. the soil column is assumed nondeformable) by

$$L = \bigcup_{i=1}^{n} \ell_i \qquad (25)$$

where ℓ_i is the length of each element. The state variable is approximated within each finite element by the Ritz procedure

$$u(x, t) = \sum N_j(x, t)u_j \qquad (26)$$

where N_j is the appropriate linearly independent shape functions; u_j is the state variable values ($u = \phi$ or T depending on the equation being solved) at element-nodal points designated by the general summation index j.

The Galerkin technique utilizes the set of shape functions as the weighting functions, which indicates that the corresponding finite element representation for the infiltration process for $\partial u/\partial t$ assumed invariant, is

$$\sum_{i=1}^{n} \int_{\ell_i} A(u)\ N_j\ dx = 0 \qquad (27)$$

where A is the partial differential operator, given in eqs. 3 and 22, operating on the respective state variable, designated by U. Equation 27 is customarily integrated by parts as described in Desai (4) yielding a statement of the boundary conditions and another integral. This integral is solved by substituting the appropriate element approximations and shape functions into the integrand and solving by numerical integration. The nonlinear nature of the partial differential equations, however, generally introduces difficulties in integrating. It is customary to deal with this problem by assuming the parameters to be constant within each finite element during a finite time interval, Δt (e.g. Guymon and Luthin (12)). The Crank-Nicholson time advancement approximation has been widely used (Desai (4)) to solve the time derivative.

The Crank-Nicholson formulation reduces eq. 27, where parameters are assumed constant within each finite element, into a system of linear equations expressed in matrix form as

$$\{\underset{\sim}{P} + \frac{\Delta t}{2}\underset{\sim}{S}\}\underset{\sim}{u}^{i+1} = \{\underset{\sim}{P} - \frac{\Delta t}{2}\underset{\sim}{S}\}\underset{\sim}{u}^{i} \qquad (28)$$

where $\underset{\sim}{P}$ is a capacitance matrix and is a function of element nodal global coordinates; $\underset{\sim}{S}$ is a stiffness matrix and is a function of element nodal global coordinates and constant finite element parameters (during time step Δt); Δt is the finite time step increment; and $\underset{\sim}{u}^k$ is the vector of nodal state variable approximations at time steps $k = i, i + 1, \cdots$.

By assuming the parameter function to be constant within each finite element, error is introduced into the numerical solution. Additional error is involved by employing the Crank-Nicholson finite difference time advancement. Hromadka and Guymon (14) studied a variety of methods for determining quasi-constant values of the parameter functions (e.g. $K(\psi)$) used in the linearized formation of eq. 28. The unmodified Crank-Nicholson capacitance matrix based upon values of the parameters in eqs. 3 and 22 evaluated as the mean parabolic value of the state variable gave superior results and are the basis of our numerical approach. Although there is no numerical advantage to using shape functions of higher than first order when employing Crank-Nicholson, we use a quadratic (parabolic) shape function to facilitate estimation of nonlinear parameters and secondary variables (e.g. velocity flux) required in the numerical solution procedure.

The isothermal heat budget method is used as described by eq. 15. The liquid water sink approximation as described by eq. 21 is also used in the model in order to carefully balance mass at each time step. Computed hydraulic heads are adjusted in accordance with the moisture sink term. At each time step, hydraulic conductivity is estimated by eq. 20, θ^* is estimated by eq. 16, and heat capacity is estimated by eq. 2.

Initial conditions for temperature and hydraulic head are required as defined by eqs. 17 and 24. The upper and lower temperature boundary conditions must be specified according to eq. 17. The lower hydraulic head boundary conditions must be specified and is usually the water table which may be a function of time. The upper hydraulic head boundary condition, specified by eq. 24, is determined by a special algorithm. The Crank-Nicholson procedure results in bothersome oscillatory behavior at the soil surface element during the early simulation phase. Computed hydraulic heads at any Δt time step in the top element are smoothed by fitting a parabola through the computed heads and forcing the parabola slope to be zero at $x=0$, insuring no liquid flux from the column at its top. If the top of the column is freezing, or any lower element for that matter, a specified unfrozen water content as described by Anderson, et al (1) is assumed, and using the fact $\phi=\psi-x$ and eq. 19, hydraulic head is recomputed. This procedure essentially means that a moving boundary condition is used and avoids the considerable problem of defining the soil surface boundary condition.

The computer program is written in FORTRAN IV and has been designed to run on small to medium size computers such as the PDP 11/34. To conserve space and computer execution time, full advantage has been taken of the banded nature of the stiffness and capacitance arrays.

APPLICATION OF MODEL

The proposed model was applied to a set of laboratory data as described by Berg et al (2) with excellent results. Frost heave measured in a labaratory column was simulated with the model using experimental data on soil parameters and boundary conditons. Table 1 shows a comparison of measured and simulated unrestrained frost heave, position of the freezing isotherm, and unfrozen pore pressures at 24 cm below the column top. The simulation was discontinued after 15 days; however, other computer trials indicated that frost heave estimates are very stable, and frost heave can be realistically simulated for much longer periods of time.

Comparison of measured and simulated restrained frost heave were performed for one case which was for a laboratory column test with a 5 psi surcharge added to the soil column top. Table 2 shows the comparison for simulated and measured frost heave for 20 days. Surcharge effects are modeled by computing the total weight of soil water, and ice above the freezing front, including the surcharge. This weight is converted to an equivalent water pressure which is added to the computed water tension at the freezing front. This procedure effectively reduces the water tension at the freezing front which results in less water being drawn into the freezing area. The model accurately predicts the start of heave which occurred seven days after the initiation of the freezing test. Simulated heaves agree well with measured results for the 20-day simulation. However, after about 23 days, heave seemed to level off in the laboratory column while simulated heaves were greater, somewhat tending to diverge. We attribute this to inaccurate representation of boundary conditions which will be corrected in subsequent column tests.

TABLE 1

COMPARISON OF SIMULATED AND RECORDED FROST HEAVE
UNRESTRAINED CASE

	TIME, DAYS		
	5	10	15
Laboratory Column			
Total Frost Heave, cm	1.6	2.8	4.0
0°C Isotherm Depth, cm	6	11	11
Tension at 24 cm Depth, cm of water	~200	~200	~200
Simulated with Model			
Total Frost Heave, cm	1.5	2.9	3.9
0°C Isotherm Depth cm	4.5-7.5	7-10	10-12.5
Tension at 24 cm	110	130	150

TABLE 2

COMPARISON OF SIMULATED FROST HEAVE AND MEASURED FROST HEAVE, RESTRAINED CASE (5 PSI SURCHARGE)

	5 Days	10 Days	15 Days	20 Days
Laboratory Column, cm	0.00	0.68	1.50	2.28
Simulated with Model, cm	0.00	0.54	1.43	2.30
Error, %	0	21	5	1

DISCUSSION

Results of our model indicate that the approach we use is feasible for estimating frost heave. The macroscopic thermodynamic approach for estimating ice segregation and accounting for freezing moisture is practical and minimizes the need for a detailed understanding of the microphysics of the ice segregation process.

Limited work on sensitivity of the many parameters that arise in the model suggest that the model is highly sensitive to hydraulic parameters and boundary conditions. This is not surprising since it has been known for some time that slight variations in soil parameters will substantially alter frost heave of a soil. It is our view that the direction future research should take is not only to develop other or better numerical algorithms but to deal with parameter identification and optimization problems and to incorporate some sort of confidence limits to modeled solutions in order to apply frost heave models to field situations.

REFERENCES

1. D.M. Anderson, A.R. Tice, and H.L.McKim, The unfrozen water and the apparent specific heat capacity of frozen ground, Second International Permafrost Conference, Yakutsk, U.S.S.R., North Amer. Cont., Nat. Acad. of Sci., pp. 289-294, 1973.
2. R.L. Berg, J. Ingersoll, and G.L. Guymon, Frost heave in an instrumented soil column, 1979 Conf. on Soil-Water Problems in Cold Regions, Calgary, Alberta, Canada, 1979.
3. B.J. Dempsey, Mathematical modeling of simultaneous transport of moisture and heat in soils with emphasis on hydraulic properties and input data. Presented at FHWA Annual Research Review, Atlanta, Georgia, October 1977.
4. C.S. Desai, Elementary finite element method, Prentice-Hall, Inc., 1979.
5. D.A. DeVries, Thermal properties of soils, Physics of Plant Environment (ed. W.E. Van Wijk), North-Holland Pub. Co., Amsterdam, pp. 210-235, 1966.
6. C. Dirksen and R.D. Miller, Closed system freezing of unsaturated soil, Proceedings, Soil Sciences Society of America, Vol. 30 pp. 168-173, 1966.
7. J-M.S. Dudek and J.T. Holden, A theoretical model for frost heave, Int. Conf. on Numerical Methods in Thermal Problems, Swansea, England, 1979.
8. W.R. Gardner, Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table, Soil Sci. 85, pp. 228-232, 1958.
9. P.H. Groenevelt and B.D. Kay, Water and ice potentials in frozen soils, Water Resources Res., 13(2), pp. 445-449, 1977.

10. G.L. Guymon and R.L. Berg, Galerkin finite element analog of coupled moisture and heat transport in algid soils, First Int. Conf. on Finite Elements in Water Resources, Princeton University, 3.107-3.114, July 1976.
11. G.L. Guymon and T.V. Hromadka, II, Finite element model of transient heat conduction with isothermal phase change (two and three dimensional) U.S. Army Cold Regions Research and Engineering Laboratory, Special Report 77-38, 1977.
12. G.L. Guymon and J.N. Luthin, A coupled heat and moisture transport model for arctic soils, Water Resources Res., 10(5), pp. 995-1001, 1974.
13. R.L. Harlan, Analysis of coupled heat-fluid transport in partially frozen soil, Water Resources Res., 9(5), 1314-1323, 1973.
14. T.V. Hromadka and G.L. Guymon, Some effects of linearizing the unsaturated soil-moisture transfer diffusion model, Manuscript Submitted to Water Resources Research, 1979.
15. C.T. Hwang, Frost heave design of a chilled gas pipeline, 30th Canadian Geotechnical Conference, Saskatoon, Sask., Canada, 1977.
16. Y-W Jame, Heat and mass transfer in freezing unsaturated soil, Ph.D. dissertation, Univ. of Saskatchewan, Saskatoon, Canada, 1978.
17. Y-W Jame and D.I. Norum, Heat and mass transfer in freezing unsaturated soil in a closed system, Proc. 2nd Conference on Soil Water Problems in Cold Regions, Edmonton, Alberta, Canada, 1976.
18. S. Kinosita, Soil-water movement and heat flux in freezing ground, Proc. Conf. on Soil Water Problems in Cold Regions, Calgary, Alberta, Canada, 1975.
19. Y. Nakano and J. Brown, Effect of a freezing zone on finite width on the thermal regime of soils, Water Resources Res., 7(5), pp. 1226-1233, 1971.
20. S.I. Outcalt, A simple energy balance model of ice segregation, 1979 Conf. on Soil Water Problems in Cold Regions, Calgary, Alberta, Canada, 1979.
21. Marsha I. Sheppard, B.O. Kay and J.P.G. Loch, Development and testing of a computer model and for heat and mass flow in freezing soils, Third International Permafrost Conf., Edmonton, Canada, 1977.
22. S. Takagi, The adsorption force theory of frost heaving, U.S. Army Cold Regions Research and Engineering Laboratory Manuscript, pp. 78, 1979.
23. G.S. Taylor and J.N. Luthin, A model for coupled heat and moisture transfer during soil freezing, Canadian Geotechnical Journal, 15, pp. 548-555, 1978.