Nodal domain integration model of two-dimensional advection-diffusion processes

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The nodal domain integration method is applied to a two-dimensional advection-diffusion process in an anisotropic inhomogeneous medium. The domain is discretised into the union of irregular triangle finite elements with vertex-located nodal points and a linear trial function is used to approximate the governing flow equation's state variable in each element. Non-linear parameters are assumed quasi-constant for small durations in time in each element. The resulting numerical model represents the Galerkin and subdomain integration weighted residual methods and the integrated finite difference method as special cases. Both Dirichlet and Neumann boundary conditions are accommodated in a manner similar to the Galerkin finite element approach.

INTRODUCTION

Optimum numerical methods for the solution of the time dependent partial differential equations such as occur in the theory of diffusion processes and advection-diffusion processes in porous media flow are the subject of a substantial body of literature.[1,2] The overwhelming preference in the literature is towards the finite difference and finite element numerical approaches. In comparison between the finite element and finite difference methods, three main advantages are generally cited in the literature for preferring the finite element technique to the finite difference one (although several cases exist which determine identical numerical analogs), namely

(i) zero-flux type boundary conditions (i.e. Neumann) are handled 'naturally' without the need for special flux-boundary approximators as required in the current finite difference methods;
(ii) the size and shape of finite elements can be variable throughout the solution domain;
(iii) inhomogeneous and/or anisotropic domains are easily accommodated by a finite element analog.

In this paper it will be shown that by the proper definition of nodal domains as the set of set-intersections of a finite element and subdomain discretisation of the global domain, a finite-element matrix system can be determined for a finite difference or subdomain integration numerical analog similar to the Galerkin finite element matrix systems. These resulting element matrix systems will also be shown to satisfy Neumann and Dirichlet boundary conditions identical to the Galerkin numerical analog, and also accommodate anisotropic and inhomogeneous domain characteristics.

Additionally, it will be shown that for a linear trial function estimate of the state variable in each finite element, the resulting element matrix systems for the various numerical methods considered can be represented by a single nodal domain integration finite-element matrix system by the appropriate specification of a single constant parameter.

A two-dimensional advection-diffusion process in an inhomogeneous anisotropic medium is examined. The finite element configuration assumed used to discretise the global domain and boundary is the triangle element with three vertex-located nodal points (linear trial function assumed in each finite element).

Since the triangle-element configuration has been studied for an isotropic inhomogeneous diffusion process in a previous paper[3] only the anisotropy of the medium and the inclusion of (x, y) advection components will be addressed in this paper.

GOVERNING EQUATIONS

A two-dimensional advection-diffusion process in an inhomogeneous anisotropic non-deformable medium without sources or sinks may be generally expressed by the partial differential equation:

$$\frac{\partial}{\partial x}\left[K_x\frac{\partial T}{\partial x}-UT\right]+\frac{\partial}{\partial y}\left[K_y\frac{\partial T}{\partial y}-WT\right]=C\frac{\partial T}{\partial t},\quad (x,y)\in\Omega \tag{1}$$

where (x, y) are spatial coordinates in global domain Ω; t is time; T is the state variable (e.g. temperature, or concentration of a conservative specie); U and W are the (x, y) related advection components (e.g. fluid velocity); $(K_x = K_{xx}, K_y = K_{yy})$ are assumed principal axis values of conductivity (e.g. heat conductivity or Fickian dispersion; and C is a capacitance coefficient (e.g. volumetric heat capacity). It is assumed in subsequent model development that:

$$\zeta=\zeta(x,y,t),\qquad \text{for}\qquad \zeta\in\{K_x,K_y,U,W,C,T\} \tag{2}$$

In vector notation, (1) may be written as:

$$\int_\Gamma \vec{q}\cdot d\vec{\Gamma}=\int_\Omega C\frac{\partial T}{\partial t}\,dA \tag{3}$$

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where Γ is the boundary of Ω; $\vec{d\Gamma}$ is the outward unit normal vector to Γ, $\|d\vec{\Gamma}\| = d\Gamma$; and

$$\vec{q} \equiv \left(K_x \frac{\partial T}{\partial x} - UT\right)\vec{i} + \left(K_y \frac{\partial T}{\partial y} - WT\right)\vec{j} \qquad (4)$$

For an arbitrary n-density nodal point distribution in Ω with associated subdomains R_m with boundaries B_m, the following subdomain definitions are assumed:

$$\Omega \equiv \bigcup_{m=1}^{n} R_m \qquad (5)$$

i.e. Ω is the union of subdomains R_m;

$$R_m \equiv \bar{R}_m = R_m \cup B_m \qquad (6)$$

i.e. each domain is defined as the union of domain and boundary;

$$R_m \cap R_k \equiv B_m \cap B_k \qquad (7)$$

i.e. each intersection of subdomains is the intersection of boundaries, and

$$(x_m, y_m) \in R_m; \quad (x_m, y_m) \notin R_k, \quad m \neq k \qquad (8)$$

Then (3) may be replaced with the corresponding subdomain equations:

$$\int_{\Gamma} \vec{q} \cdot d\vec{\Gamma} = \int_{B_m} \vec{q} \cdot d\vec{\Gamma} \qquad (9)$$

$$\int_{\Omega} C \frac{\partial T}{\partial t} dA = \int_{R_m} C \frac{\partial T}{\partial t} dA \qquad (10)$$

For a finite element discretization elements $\hat{\Omega}_e$ are defined in Ω as follows:

$$\Omega \equiv \cup \hat{\Omega}_e \qquad (11)$$

$$\hat{\Omega}_e = \hat{\Omega}_e \cup \hat{\Gamma}_e \qquad (12)$$

A set of nodal domains Ω_j[3] is defined by

$$\Omega_j \equiv \{R_m \cap \hat{\Omega}_e\} \qquad (13)$$

A compact cover of $\hat{\Omega}_e$ is given by:

$$\hat{\Omega}_e = \cup \Omega_j, \quad j \in S_e \qquad (14)$$

where S_e is the set of subscripts defined by:

$$S_e \equiv \{j \mid \Omega_j \cap \hat{\Omega}_e \neq \{\emptyset\}\} \qquad (15)$$

A finite element matrix system equivalent to the governing domain equation (1) is generated for finite element $\hat{\Omega}_e$ by:

$$\left\{\int_{\Gamma_j} \vec{q} \cdot d\vec{\Gamma} - \int_{\Omega_j} C \frac{\partial T}{\partial t} dA\right\} = \{0\}, \quad j \in S_e \qquad (16)$$

Likewise, a subdomain integration statement for (1) is generated for subdomain R_m by combining (9) and (10):

$$\left\{\int_{B_m} \vec{q} \cdot d\vec{\Gamma} - \int_{R_m} C \frac{\partial T}{\partial t} dA\right\} = \{0\} \qquad (17)$$

Expanding the transport integral of (16) gives:

$$\int_{\Gamma_j} \vec{q} \cdot d\vec{\Gamma} = \int_{\Gamma_j} \left[K_x \frac{\partial T}{\partial x}\vec{i} + K_y \frac{\partial T}{\partial y}\vec{j} - UT\vec{i} - WT\vec{j}\right] \cdot d\vec{\Gamma} \qquad (18)$$

or

$$\int_{\Gamma_j} \vec{q} \cdot d\vec{\Gamma} = \int_{\Gamma_j \cap \hat{\Gamma}_e} \left[K_x \frac{\partial T}{\partial x}\vec{i} + K_y \frac{\partial T}{\partial y}\vec{j}\right] \cdot d\vec{\Gamma} - \int_{\Gamma_j \cap \hat{\Gamma}_e} [UT\vec{i} + WT\vec{j}] \cdot d\vec{\Gamma} + \int_{\Gamma_j - \Gamma_j \cap \hat{\Gamma}_e} \vec{q} \cdot d\vec{\Gamma} \qquad (19)$$

where $\hat{\Gamma}_e \equiv$ boundary of finite element $\hat{\Omega}_e$. The first integral in the expansion of (19) satisfies Neumann boundary conditions on $\hat{\Gamma}_e$ or preserves flux continuity (due to conduction processes) between finite elements. In the global assemblage of $\hat{\Omega}_e$, the first integral in the expansion of (19) also satisfies Neumann boundary conditions on the discretised approximation of global boundary Γ by $\hat{\Gamma}_e$. From (19), the element matrix system of (16) is given by:

$$\left\{\int_{\Gamma_j - \Gamma_j \cap \hat{\Gamma}_e} \vec{q} \cdot d\vec{\Gamma} - \int_{\Gamma_j \cap \hat{\Gamma}_e} [UT\vec{i} + WT\vec{j}] \cdot d\vec{\Gamma} - \int_{\Omega_j} C \frac{\partial T}{\partial t} dA\right\} = \{0\}, \quad j \in S_e \qquad (20)$$

NUMERICAL MODEL

For a finite element cover $\hat{\Omega}_e$ (of global domain Ω) composed of triangle elements with three vertex located nodal points,[1] the state variable T is assumed approximated within each $\hat{\Omega}_e$ by a linear trial function:

$$T(x, y, t) \simeq \hat{T}_e(x, y, t), \quad (x, y) \in \hat{\Omega}_e \qquad (21)$$

where

$$\hat{T}_e(x, y, t) \equiv L_j(x, y)\, T_j(x_j, y_j, t) \qquad (22)$$

where the L_j are the usual area shape local coordinates

$$L_j \equiv \frac{A_j}{A^e} \qquad (23)$$

and T_j are nodal values of $\hat{T}_e$ at nodal points j; and A^e is the area of triangle finite element $\hat{\Omega}_e$.

Galerkin finite element analog

For a linear trial function $\hat{T}_e$ in finite element $\hat{\Omega}_e$, the Galerkin method of weighted residuals approximates (1) in $\hat{\Omega}_e$ by:

$$\int_{\hat{\Omega}_e} \left\{\frac{\partial}{\partial x}\left[K_x \frac{\partial \hat{T}_e}{\partial x} - U\hat{T}_e\right] + \frac{\partial}{\partial y}\left[K_y \frac{\partial \hat{T}_e}{\partial y} - W\hat{T}_e\right] - C\frac{\partial \hat{T}_e}{\partial t}\right\} \times L_j \, dA = \{0\} \qquad (24)$$

Solving (24) generates an order 3 element matrix system when (24) is quasi-linearised by assuming non-linear parameters constant for small timestep Δt;[2] thus (24) is quasi-linearised by:

$$\iint_{\hat{\Omega}_e} \left\{ \frac{\partial}{\partial x}\left[K_x^e \frac{\partial \hat{T}_e}{\partial x} - U^e \hat{T}_e\right] + \frac{\partial}{\partial y}\left[K_y^e \frac{\partial \hat{T}_e}{\partial y} - W^e \hat{T}_e\right] - C^e \frac{\partial \hat{T}_e}{\partial t}\right\} L_j \, dA = \{0\} \quad (25)$$

where it is assumed that parameters $\zeta(0 \leqslant t \leqslant \Delta t) = \zeta^e + \zeta(t)$, and $(\partial/\partial t)\,\zeta(t) \cong 0$.

From (22):

$$\frac{\partial \hat{T}_e}{\partial x} = \left(\Sigma \frac{\partial L_j}{\partial x} T_j\right) = \frac{1}{2A^e}\left[y_{32}T_1 + y_{13}T_2 + y_{21}T_3\right] \quad (26)$$

which is a constant in $\hat{\Omega}_e$, and $y_{ij} \equiv y_j - y_i$.

The Galerkin (x, y) convective term components of (25) are given by

$$\mathbf{C}_x^e + \mathbf{C}_y^e \equiv \iint_{\hat{\Omega}_e} \left\{\frac{\partial}{\partial x}(U^e \hat{T}_e) + \frac{\partial}{\partial y}(W^e \hat{T}_e)\right\} L_j \, dA \quad (27)$$

For the linear trial function assumption:

$$\mathbf{C}_x^e + \mathbf{C}_y^e = \left(U^e \frac{\partial \hat{T}_e}{\partial x} + W^e \frac{\partial \hat{T}_e}{\partial y}\right) \int_{\hat{\Omega}_e} L_j \, dA \quad (28)$$

Combining (26) and (28) gives:

$$\mathbf{C}_x^e + \mathbf{C}_y^e = \frac{U^e}{6}\{y_{32}T_1 + y_{13}T_2 + y_{21}T_3\}^e + \frac{W^e}{6} \times \{x_{23}T_1 + x_{31}T_2 + x_{12}T_3\}^e \quad (29)$$

The Galerkin conduction term components of (25) are given by:

$$\iint_{\hat{\Omega}_e} \left\{\frac{\partial}{\partial x}\left(K_x^e \frac{\partial \hat{T}_e}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y^e \frac{\partial \hat{T}_e}{\partial y}\right)\right\} L_j \, dA \quad (30)$$

which after integration by parts gives:[1]

$$-\iint_{\hat{\Omega}_e} \left\{K_x^e \frac{\partial \hat{T}_e}{\partial x}\frac{\partial N_j}{\partial x} + K_y^e \frac{\partial \hat{T}_e}{\partial y}\frac{\partial N_j}{\partial y}\right\} dA \quad (31)$$

where in (31) Neumann or specified boundary conditions are assumed on global blundary Γ, and conduction flux continuity is assumed between $\hat{\Omega}_e$.

Equation (31) results in the symmetrical element conduction matrix system:

$$\mathbf{K}_x^e \mathbf{T}^e + \mathbf{K}_y^e \mathbf{T}^e \equiv$$

$$\frac{K_x^e}{4A^e}\begin{bmatrix} (y_{23}^2) & -(y_{13}y_{23}) & (y_{12}y_{23}) \\ & (y_{13}^2) & -(y_{12}y_{13}) \\ \text{Symmetrical} & & (y_{12}^2)\end{bmatrix}^e \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix}^e + \frac{K_y^e}{4A^e}\begin{bmatrix} (x_{23}^2) & -(x_{13}x_{23}) & (x_{12}x_{23}) \\ & (x_{13}^2) & -(x_{12}x_{13}) \\ \text{Symmetrical} & & (x_{12}^2)\end{bmatrix}^e \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix}^e \quad (32)$$

where

$$A^e = \int_{\hat{\Omega}_e} dA$$

Finally, the Galerkin capacitance term of (25) is determined from:

$$\left\{\int_{\hat{\Omega}_e} C^e \frac{\partial \hat{T}_e}{\partial t} L_j \, dA\right\} \quad (33)$$

which can be simplified as (for C^e quasi-constant during small timestep Δt)

$$C^e\left\{\int_{\hat{\Omega}_e} \frac{\partial \hat{T}_e}{\partial t} L_j \, dA\right\} \quad (34)$$

Solution of (34) results in the symmetrical element capacitance matrix:

$$\mathbf{P}^e(2) = \frac{C^e A^e}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial T_1}{\partial t} \\ \dfrac{\partial T_2}{\partial t} \\ \dfrac{\partial T_3}{\partial t} \end{Bmatrix}^e \quad (35)$$

The resulting element matrix system approximation of (1) on $\hat{\Omega}_e$ can be written in the form:

$$(\mathbf{K}_x^e \mathbf{T}^e + \mathbf{K}_y^e \mathbf{T}^e) + (\mathbf{C}_x^e \mathbf{T}^e + \mathbf{C}_y^e \mathbf{T}^e) + \mathbf{P}^e(2)\frac{\partial \mathbf{T}^e}{\partial t} = \{0\} \quad (36)$$

Where the $(\mathbf{K}_x^e + \mathbf{K}_y^e)$, $(\mathbf{C}_x^e + \mathbf{C}_y^e)$ and $\mathbf{P}^e(2)$ matrices are given by (32), (29) and (35), respectively.

Assemblage of the finite element matrix contributions into a global matrix system generates the Galerkin finite element numerical approximation of (1) in Ω for an assumed linear trial function $\hat{T}_e$ in each $\hat{\Omega}_e$. By definition, the global system satisfies both Neumann and Dirichlet boundary conditions on global boundary, Γ.

Subdomain integration analog

The subdomain version of the weighted residuals approach approximates (1) in Ω by:[1]

$$\iint_{\Omega} \left(\frac{\partial}{\partial x}\left[K_x \frac{\partial T}{\partial x} - UT\right] + \frac{\partial}{\partial y}\left[K_y \frac{\partial T}{\partial y} - WT\right] - C\frac{\partial T}{\partial t}\right) \times N_j \, dA = 0 \quad (x, y) \in \Omega \quad (37)$$

where

$$N_j \equiv \begin{cases} 1, (x, y) \in R_j \\ 0, \text{otherwise} \end{cases} \quad (38)$$

and

$$\Omega = \bigcup_{i=1}^{n} R_j \quad (39)$$

From (19) and (20), (37) may be rewritten with respect to the finite element cover $\hat{\Omega}_e$ of Ω as (Fig. 1):

$$\int_{\Gamma_j - \Gamma_j \cap \hat{\Gamma}_e} \vec{q} \cdot d\vec{\Gamma} - \int_{\Gamma_j \cap \hat{\Gamma}_e} [UT\vec{i} + WT\vec{j}] \cdot d\vec{\Gamma} = \int_{\Omega_j} C \frac{\partial T}{\partial t} \, dA \quad j \in S_e \quad (39)$$

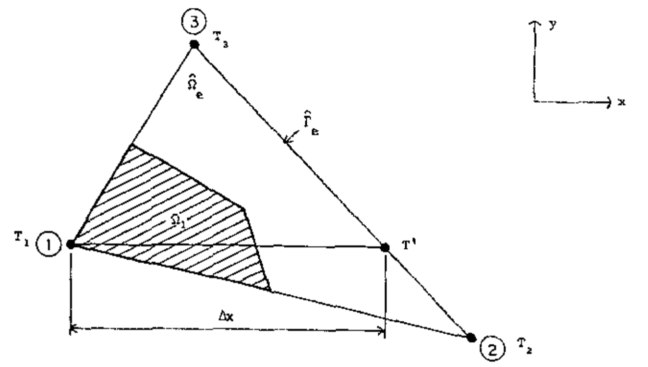

Figure 1. Nodal domain integration – model geometry in triangle finite element $\hat{\Omega}_e$

where the set of nodal domains Ω_j provides a cover of each triangle finite element $\hat{\Omega}_e$ as defined by the partition of $\hat{\Omega}_e$ from the triangle medians.[3] Equation (39) satisfies both Dirichlet and Neumann boundary conditions similar to the Galerkin approach.

For a linear trial function $\hat{T}_e$ in $\hat{\Omega}_e$, the x-direction convective term is determined from (37) and (39) by:

$$\int_{\Omega_j} U^e \frac{\partial \hat{T}_e}{\partial x} \mathrm{d}A = U^e \frac{\partial \hat{T}_e}{\partial x} \int_{\Omega_j} \mathrm{d}A = U^e \frac{\partial \hat{T}_e}{\partial x} \frac{A^e}{3}, \quad j \in S_e \tag{40}$$

where $\partial \hat{T}_e / \partial x$ is constant in $\hat{\Omega}_e$.

Equation (40) is evaluated with the aid of Fig. 1 where an intermediate domain boundary variable T' is given as:

$$T' = T_2 + \frac{y_{12}}{y_{23}}(T_2 - T_3) \tag{41}$$

which upon rearranging is:

$$(y_{32})(T' - T_1) = y_{23} T_1 + y_{31} T_2 + y_{12} T_3 \tag{42}$$

Comparing (40) and (42) gives:

$$\int_{\Omega_j} U^e \frac{\partial \hat{T}_e}{\partial x} \mathrm{d}A = U^e \frac{(T' - T_1)}{\Delta x} \frac{\Delta x (y_{32})}{6} \tag{43}$$

Therefore, the subdomain integration model gives:

$$\int_{\Omega_j} U^e \frac{\partial \hat{T}_e}{\partial x} \mathrm{d}A = \frac{U^e}{6}\{y_{32}T_1 + y_{13}T_2 + y_{21}T_3\}, \quad j \in S_e \tag{44}$$

where in (43) the product $\Delta x(y_{32}) = -2A^e$.

Comparison of (44) to (29) shows that the Galerkin and subdomain integration numerical approaches determine identical x-direction convective term element matrices, $\mathbf{C}_x^e$. The y-direction convective term element matrix $\mathbf{C}_y^e$ is derived analogous to the above. The anisotropic conduction term (linear trial function approximation $\hat{T}_e$ for T in $\hat{\Omega}_e$) determines a symmetrical element conduction matrix system identical to (32). Finally, Hromadka *et al.*,[3] determine the subdomain finite element capacitance matrix in $\hat{\Omega}_e$ for a linear trial function $\hat{T}_e$ as:

$$\int_{\Omega_j} C^e \frac{\partial \hat{T}_e}{\partial t} \mathrm{d}A = \frac{C^e A^e}{108} \begin{bmatrix} 22 & 7 & 7 \\ 7 & 22 & 7 \\ 7 & 7 & 22 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial T_1}{\partial t} \\ \dfrac{\partial T_2}{\partial t} \\ \dfrac{\partial T_3}{\partial t} \end{Bmatrix}^e \tag{45}$$

The finite element $\hat{\Omega}_e$ contribution to the solution of (1) on global domain Ω is given by:

$$(\mathbf{K}_x^e \mathbf{T}^e + \mathbf{K}_y^e \mathbf{T}^e) + (\mathbf{C}_x^e \mathbf{T}^e + \mathbf{C}_y^e \mathbf{T}^e) + \mathbf{P}^e(22/7) \frac{\partial \mathbf{T}^e}{\partial t} = \{0\} \tag{46}$$

Finite difference analog

Hromadka *et al.*,[3] show that the integrated finite difference solution[4] of (1) without advection is identical to the linear trial function subdomain integration approach except that the element capacitance term is given by:

$$\mathbf{P}^e(\infty) = \frac{C^e A^e}{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{47}$$

Thus, the finite difference numerical approximation for equation (1) in $\hat{\Omega}_e$ according to Spalding[4] is given by:

$$(\mathbf{K}_x^e \mathbf{T}^e + \mathbf{K}_y^e \mathbf{T}^e) + (\mathbf{C}_x^e \mathbf{T}^e + \mathbf{C}_y^e \mathbf{T}^e) + \mathbf{P}^e(\infty) \frac{\partial \mathbf{T}^e}{\partial t} = \{0\} \tag{48}$$

In this case, it is important to note that the finite difference global matrix system resulting from the combination of all element matrix contributions satisfies both Neumann and Dirichlet boundary conditions according to (19) and (20).

Nodal domain integration

Hromadka *et al.*,[5] develop an extension of the subdomain integration version of the weighted residuals process resulting in an element matrix system for approximation of (1) in $\hat{\Omega}_e$ similar to (36), (46) and (48) except that the element capacitance matrix system is defined by:

$$\mathbf{P}^e[\bar{\eta}(t)] = \frac{C^e A^e}{3(\bar{\eta}(t) + 2)} \begin{bmatrix} \bar{\eta}(t) & 1 & 1 \\ 1 & \bar{\eta}(t) & 1 \\ 1 & 1 & \bar{\eta}(t) \end{bmatrix} \tag{49}$$

where $\bar{\eta}(t)$ is defined by:

$$\bar{\eta}(t) \equiv 1/3 \, \Sigma \eta_j(t) \quad j \in S_e \tag{50}$$

in order to preserve symmetry in $\mathbf{P}^e[\bar{\eta}(t)]$. It was suggested that $\bar{\eta}(t)$ be varied between elements $\hat{\Omega}_e$ and with respect to time. Some methods of computing $\bar{\eta}_j(t)$ are examined for a one-dimensional advection-diffusion problem in Hromadka and Guymon.[5] Using (49), the nodal domain integration model for (1) in $\hat{\Omega}_e$ may be written as:

$$(\mathbf{K}_x^e \mathbf{T}^e + \mathbf{K}_y^e \mathbf{T}^e) + (\mathbf{C}_x^e \mathbf{T}^e + \mathbf{C}_y^e \mathbf{T}^e) + \mathbf{P}^e[\eta(t)] \frac{\partial \mathbf{T}^e}{\partial t} = \{0\} \tag{51}$$

where $\bar{\eta}(t) = (2, 22/7, \infty)$ gives the Galerkin finite element, subdomain integration, and finite difference numerical approximations of (36), (46) and (48), respectively. Similar element matrix systems have been developed for solution of (1) on irregular one-dimensional elements[6] and irregular two-dimensional rectangular elements;[7] for both finite element configurations, $\bar{\eta}(t) = (2, 3, \infty)$ gave Galerkin finite element, subdomain integration, and finite difference numerical approximations for an assumed linear trial function within each finite element.

CONCLUSIONS

The nodal domain integration model for the approximation of two dimensional advection–diffusion processes in a inhomogeneous anisotropic medium is developed. Using a triangle element, the global domain is discretised into a finite element domain and boundary approximation, and a linear trial function is used to approximate the state variable in each finite element. A finite difference analog is developed according to Spalding,[4] and a Galerkin and subdomain weighted residuals analog are developed according to Pinder and Gray.[1] Comparison of these numerical analogs to the resulting nodal domain integration method indicates that the Galerkin, subdomain integration, and finite difference methods can be represented by the nodal domain integration numerical statement in each finite element by the specification of a single constant parameter in the nodal domain integration element matrix system. The resulting nodal domain integration analog (and accordingly the finite difference and subdomain integration numerical analogs) is shown to represent both Neumann and Dirichlet boundary conditions on the global boundary similar to the Galerkin weighted residuals analog.

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