Reprinted from:
Permafrost: Fourth International Conference, Proceedings
ISBN 0-309-03435-3
National Academy Press
Washington, D.C. 1983

COMPARISON OF TWO-DIMENSIONAL DOMAIN AND BOUNDARY INTEGRAL GEOTHERMAL MODELS WITH EMBANKMENT FREEZE-THAW FIELD DATA

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The time- and position-dependent locations of the 0°C isotherm were calculated using two modelling strategies: a domain method and a boundary integral method. Simulations were made for the runway embankment at Deadhorse Airport near Prudhoe Bay, Alaska. The same thermal properties, initial conditions, and boundary conditions were used in both models. Sinusoidal surface temperature variations, dependent upon surface type and exposure, were used in the simulations rather than measured surface temperatures. The positions of the 0°C isotherm determined by the boundary integral method near the time of maximum thaw penetration were essentially the same as those determined by the finite element method, and results from both models agreed closely, within a few centimeters over a total freezing depth of about 2.5 m, with the measured positions. The largest differences between measured and computed positions occurred early in the freezing and thawing seasons. The primary advantage of using the boundary integral method for problems specifically of the type considered herein is that it requires only a few nodal points, so computer simulations can be completed rapidly on a micro computer. If the two-dimensional thermal regime is necessary, the finite element method is most suitable.

During the 1977 and 1978 thawing seasons, the runway at Deadhorse Airport near Prudhoe Bay, Alaska, was improved and paved with an asphaltic concrete pavement. With cooperation from the State of Alaska Department of Transportation and Public Facilities and the Federal Aviation Administration (FAA), USACRREL installed temperature sensors beneath and adjacent to the runway. Subsurface temperatures at some locations are measured manually in liquid-filled access tubes and others are monitored automatically by a data collection platform (Berg and Barber 1982) that transmits information to USACRREL via the ERTS satellite system (McKim et al. 1975). The equipment has been in operation since August 1978.

Two modelling strategies were used to calculate seasonal thaw penetration: (a) a finite element domain method (Guymon and Hromadka 1982) and (b) a boundary integral equation method (Hromadka and Guymon 1982). The time-dependent location of the phase-change isotherm was calculated using both models. In addition, temperature variations at selected positions within the runway embankment were computed using the finite element method. Comparison of calculated temperatures and predicted 0°C isotherm locations with measured data indicate that both numerical modeling approaches are accurate tools in predicting soil thermal response in freezing/thawing environments.

DESCRIPTION OF MODELS

The domain method approximates the well known two-dimensional heat transport equation, which for a freezing or thawing soil is:

$$\frac{\partial(K_x \partial T/\partial x)}{\partial x} + \frac{\partial(K_y \partial T/\partial y)}{\partial y} = C_m \frac{\partial T}{\partial t} - L \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} + C_w \left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}\right) \quad (1)$$

where x, y = cartesian coordinates
t = time
T = temperature
K_x, K_y = the thermal conductivity of the soil-water-ice mixture
C_m = volumetric heat capacity of the soil-water-ice mixture
L = the volumetric latent heat of fusion of bulk water
ρ_i = ice density
ρ_w = water density
C_w = the volumetric heat capacity of bulk water
v_x, v_y = velocity flux components
θ_i = the volumetric ice content of the soil.

The density parameters are relatively precise for modeling purposes. Although the latent heat parameter is a function of temperature and salinity (Anderson et al. 1973), all of the water is assumed to freeze at 0°C in these simulations. C_w may be regarded as a well defined constant, but the appropriate thermal conductivity and heat capacity of the soil-water-ice mixture are not exactly known and must be estimated. DeVries' (1966) weighting method for estimating these parameters is often employed; i.e.

$$\beta = \Sigma\beta_n P_n \tag{2}$$

where β is the required parameter (K_j or C_m), P_n is the volumetric content of a specific constituent, and n indicates the nth constituent. The velocity flux parameters must be assumed or calculated from a coupled moisture transport model. Since this paper is concerned with heat conduction only, it will be assumed that moisture flux is negligible; consequently, the convective component of Equation 1 is assumed to be zero.

To solve Equation 1, initial and boundary conditions are needed. Initial conditions are of the form:

$$T(t=0) = T(x,y)$$

$$\theta_i(t=0) = \theta_i(x,y) \tag{3}$$

which are usually specified at discrete points in the solution domain. While boundary conditions may be of any form (e.g. a surface energy balance simulation), we will use two types:

$$\frac{\partial T}{\partial n} = 0, \; t > 0 \tag{4}$$

where n is a unit normal coordinate to the solution domain boundary, and

$$T(s) = N\,T_a(s,t), \quad t > 0 \tag{5}$$

where N is the Corps of Engineers n factor (Berg 1974), s is a tangent coordinate to the solution domain boundary, and T_a is the air temperature, which may be a function of time.

Domain Approach

Commonly used domain approaches include the finite element and finite difference methods. Hromadka et al. (1981) show that an infinity of mass lumped domain numerical analogs may be incorporated into a single matrix expression. The finite element, subdomain, and integrated finite difference schemes are represented as special cases. Depending on the subdomain of integration and the density of the state variable approximation, various domain algorithms may be obtained. These are unified into a single matrix representation called "nodal domain integration," which yields a system matrix similar to the finite element scheme:

$$K\,T + C(\eta)\,\dot{T} = F \tag{6}$$

where K is a square-banded symmetrical conduction matrix that is a function of thermal conductivity and global discretization, $C(\eta)$ is a square-banded symmetrical capacitance matrix that is a function of capacitance parameters and a mass lumping factor (η), $\dot{T}$ and T are vectors of unknown temperatures at discrete points and their temporal derivatives, respectively, and F is a load vector that is a function of the boundary temperatures. Hromadka et al. (1981) give complete details on the derivation of the matrices in Equation 6. Guymon and Hromadka (1982) use this technique to develop a two-dimensional model of coupled heat and moisture transport in freezing or thawing soils. In the analysis presented here, the n factor was set to a value such that a standard Galerkin finite element scheme was used in all domain computations.

Rather than solve Equation 1 in the form shown, the latent heat term is removed and is approximated as an isothermal process (Guymon and Hromadka 1982). Latent heat effects are simulated by a simple control volume approach. A discrete volume of soil is not allowed to reach subfreezing temperatures until the latent heat of fusion of all water in the volume is exhausted. Because this approximation makes it difficult to determine the position of the freezing or thawing isotherm, a pseudo apparent heat capacity approach is used by weighting the diagonal terms of the capacitance matrix. Only the heat transport component of the model was used in the study.

To solve the domain problem, the solution region is discretized into triangular finite elements, as shown in Figure 1. Temperatures are represented by linear shape functions within each triangular finite element.

Boundary Approach

For problems where phase-change effects dominate the solution, the temporal term in Equation 1 may be assumed to be negligible. If one assumes an isotropic, homogeneous medium, Equation 1 reduces to the Laplace equation:

$$\nabla^2 T = 0 \tag{7}$$

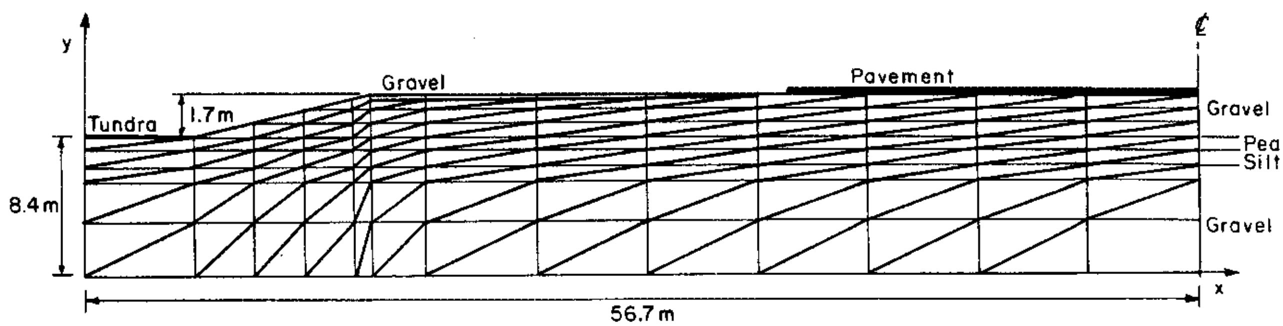

FIGURE 1 Deadhorse runway cross section showing elements and nodes used in the domain solution.

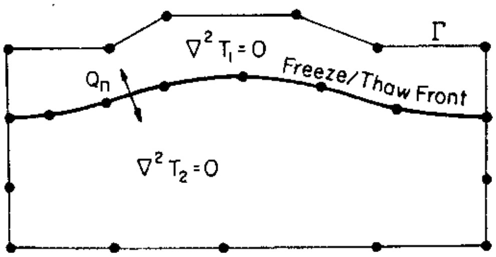

FIGURE 2 Boundaries and nodal locations for a general boundary integral solution.

where again it is assumed that moisture transport is negligible. Equation 7 is assumed to apply in both the frozen and the unfrozen regions, as shown in Figure 2.

Hromadka and Guymon (1982) have shown that boundary integral methods may be applied to geothermal problems involving soil water phase change. They applied the complex variable Cauchy integral theorem to problems where the position of the freezing or thawing isotherm is determined directly. The boundary integral approach estimates heat flux normal to a two-dimensional freezing or thawing surface. The advantage of this method is that it requires much less computer storage and execution time for certain problems than classical domain methods.

The heat flux is given by:

$$Q_x + i\,Q_y = -K\frac{\partial T}{\partial x} - i\,K\frac{\partial T}{\partial y} \qquad (8)$$

where Q_x and Q_y are heat flux and $i = \sqrt{-1}$. For any arbitrary closed-contour interior of the solution domain,

$$\int Q_n\,ds = \int Q_s\,ds = 0 \qquad (9)$$

where n and s are normal and tangential components of the contour. Equation 9 defines T as harmonic and is related to the harmonic conjugate q by the Cauchy-Riemann equations. The complex temperature is defined by

$$\xi(z) = T(x,y) + i\,q(x,y) \qquad (10)$$

where $z = x + iy$. Using Cauchy's theorem, the complex temperature on the boundary may be solved by the contour integral:

$$\xi\,(z_j) = \frac{P_v}{Si}\int\frac{\xi(z)dx}{z - z_j} \qquad (11)$$

where z_j is the nth node point on Γ. P_v indicates the Cauchy principle value, and S is the interior line segment angle at node j. Equation 11 is readily solved by assuming $\xi(z)$ is described by straight line segments between each node and by assuming $\xi(z)$ can be represented as a linear polynomial (Hromadka and Guymon 1982). Interior moving freeze/thaw boundaries are located by an isothermal change approximation:

$$L\frac{ds}{dt} = \sum_j (Q_n)_j \qquad (12)$$

where $(Q_n)_j$ is a normal heat flux component at a specific location along the phase change front during a time-step Δt. Time-steps of one day or longer may frequently be used because of the time-consuming phase change process in naturally freezing or thawing soils. The solution of Equation 11 requires known temperature or known heat flux conditions on the solution domain boundary, Γ.

The full system of equations is written in the form:

$$K(T,q) = 0 \qquad (13)$$

where K is a fully populated matrix of known coefficients and (T,q) is the vector of the complex temperature ξ_j.

FIELD DATA

Two types of data obtained at Deadhorse Airport were used in this study. The first included properties and dimensions of the soil and pavement layers. The second type included initial and boundary temperatures and the measured subsurface temperatures, which were compared with calculated subsurface temperatures.

Properties and dimensions of the soils and pavement were obtained from boring logs (Division of Aviation 1976) made prior to improving the runway and from test data developed during reconstruction of the airport (Ingersoll et al. 1979). Physical and thermal properties of the materials that were used in the thermal models are shown in Table 1. Parameters shown in Table 1 for density and water content were determined from laboratory samples. Thermal parameters were assumed from USACRREL data.

Upper boundary conditions differed depending upon the horizontal position, but they were developed from measured air temperatures in all situations. Air temperatures were adjusted by n factors (Berg et al. 1978) to obtain the surface freezing and thawing indexes and surface temperatures used in the model simulations. A sinusoidal variation of air temperature coupled with the n-factor approach was used to approximate more closely the type of analysis an engineer usually performs. Consequently, the tests of the two models presented here are conservative.

Approximately 120 thermistors are automatically monitored by a battery-powered Data Collection Platform (DCP) and data are transmitted back to USACRREL via the ERTS satellite. Each thermistor is monitored approximately once every five days, and the temperatures are stored in a computer-accessed file at CRREL. In addition, other subsurface temperature observations were obtained manually three or four times per summer. Temperature observations were plotted in a variety of graphs, i.e. thaw depth (0°C isotherm) vs time, temperatures at specified depths vs time and cross-sections of the runway at various times showing the thermal regime (isotherms).

TABLE 1 Material properties below AC pavement.

Layer	Material	Depth, m	Dry density, g/cm^3	Moisture content, % dry wt.	Thermal conductivity, cal/cm hr °C	Volumetric heat capacity, cal/cm^3 °C	Latent heat of fusion, cal/cm^3
A	Gravel	0.08–0.30	2.00	9.6	30.95	0.354	11.7
B	Gravel	0.30–2.44	1.92	7.0	29.31	0.312	6.8
C	Organic	2.44–3.05	0.80	61.6	18.90	0.400	38.4
D	Silt	3.05–4.11	1.20	50.7	17.71	0.356	46.2
E	Gravel	4.11–10.06	1.68	27.6	29.76	0.368	30.4

MODELING PROCEDURE

Both numerical modeling strategies required discretization of the domain in order to approximate inhomogeneity. In the domain method (Figure 1), 126 nodes and 210 elements were utilized. Included in the element definitions were six parameter groupings (Table 1) that incorporated the various dissimilarities of parameters and initial conditions. The boundary integral solution utilized a rescaled domain so that vertical thermal conductivity was constant above and below the 0°C isotherm (that is, frozen vs. thawed). In the rescaled domain, volumetric latent heat is adjusted to preserve the proper rescaled volumetric properties. The problem chosen is amenable to rescaling. Many heterogeneous domains are sufficiently complicated so that rescaling to arrive at a Laplacian problem is difficult or impossible. The boundary integral solution was based on 28 nodal points with eight nodal points evenly spaced along the phase-change isotherm (Hromadka and Guymon 1982).

Both models used identical specified temperature boundary conditions along the top boundary. The initial temperature distributions were inferred from measured subsurface temperatures. Zero flux conditions were assumed at the bottom and at both sides in both models.

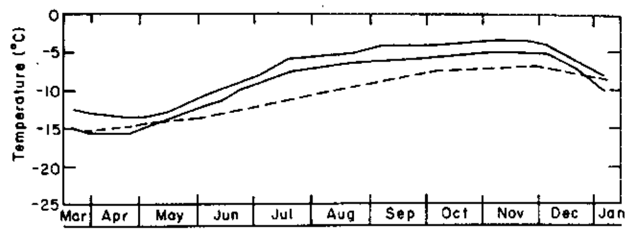

FIGURE 3 Comparison of measured temperatures (solid lines) with those computed in the domain solution (dashed line). Temperatures are approximately 4.6 m (15 ft) below the pavement surface.

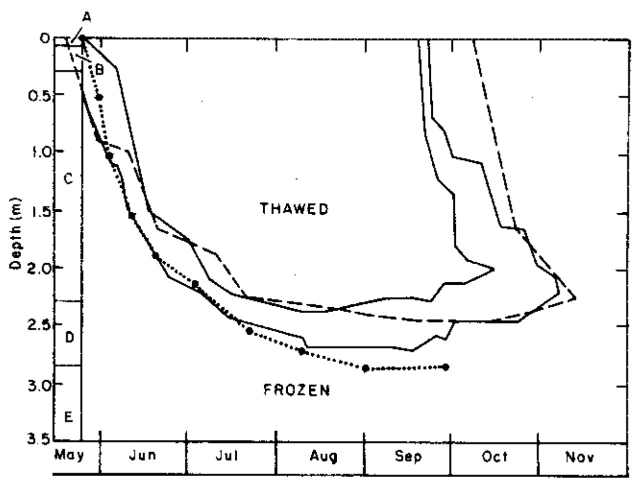

FIGURE 4 Measured (solid lines) and calculated (dashed line represents domain method and dotted line represents boundary integral method) seasonal thaw penetration. Depths beneath the pavement surface. Calculations from the domain solution.

MODEL RESULTS

Results of the domain solution are shown in Figures 3 and 4. Figure 3 shows the computed temperatures (dashed line) at a depth of 4.6 m below the pavement surface and 12.2 m from the runway centerline. Measured temperatures are shown as an envelope (solid lines) for this location. Figure 4 shows the computed thaw depths (dashed line) 12.2 m from the centerline. The measured thaw depths (solid lines) are also shown. The envelope of measured thaw depths results from several temperature assemblies at different locations beneath the pavement. Results shown for the particular embankment regime are typical of results throughout the entire embankment.

Thawing depths predicted by the two modeling approaches are given in Table 2 for modeling day number 155. These depths represent the approximate maximum thawing depths. Table 2 includes two sets of predicted thawing depths from the boundary integral approach, based on step sizes of 6 hours and 3 hours. As can be seen, both modeling approaches agree with each other quite closely. Moreover, both approaches agree well with measured thaw depths. This agreement was achieved in spite of the fact that average sinusoidal air temperature was used as a surface boundary condition in conjunction with the n-factor approach. Soil surface temperatures computed using this approach have a root mean square temperature error of at least 8°C.

In this case, the boundary integral method required signifiently less computational effort than the domain method; however, this is not a general

TABLE 2 Thawing penetration depths predicted by numerical models and measured at the airfield (model day 155).

Location (x-coordinate), m	Domain solution, m	Boundary integral solution, m Δt = 6 hrs	Δt = 3 hrs	Measured depths, m
0	0.7	0.7	0.65	0.66
6.1	0.7	0.65	0.64	0.58
7.6	1.1	2.0	2.0	---
9.1	1.7	1.0	1.0	1.09
10.4	2.0	2.3	2.3	1.98
14.0	2.2	2.1	2.2	---
26.2	2.2	2.1	2.2	---
44.5	2.4	2.6	2.7	2.72-3.02
50.6	2.4	2.8	2.7	---
56.7	2.4	2.7	2.8	2.59-3.12

* Δt equals 12.48 hrs; parameter update frequency equals 124.8 hrs.

rule since the boundary integral method has a compact matrix that will lead to less efficient computational effort in some cases. It is concluded that both models can accurately predict the thermal regime of embankments, provided thermal boundary condition and domain solution initial condition information is available.

ACKNOWLEDGMENTS

This research was sponsored by the U.S. Army Corps of Engineers and the Army Research Office under Grant No. DAAG29-79-C-0080 to the University of California, Irvine. Linda Gee, computer assistant, prepared data for the domain simulations.

REFERENCES

Anderson, D.M., Tice, A.R., and McKim, H.L., 1973, The unfrozen water and the apparent specific heat capacity of frozen ground, in Second International Permafrost Conference: Yakutsk, USSR, National Academy of Science, p. 289-294.

Berg, R.L., 1974, Energy balance on paved surface: U.S. Army Cold Regions Research and Engineering Laboratory, Technical Report No. 226.

Berg, R.L., and Barber, M.E., 1982, Prediction of long-term performance of the Deadhorse runway, U.S. Army Cold Regions Research and Engineering Laboratory, Draft Special Report.

Berg, R.L., Brown, J., and Haugen, R., 1978, Thaw penetration and permafrost conditions associated with the Livengood to Prudhoe Bay road, Alaska in Proceedings, Third International Conference on Permafrost: Edmonton, Alberta, National Research Council of Canada, pp. 615-621.

DeVries, D.A., 1966, Thermal properties of soils: Physics of Plant Environment, W.R. Van Wijk, ed.: North-Holland, Amsterdam, p. 210-235.

Division of Aviation, 1976, Engineering geology and soils report, Deadhorse subsurface soils: State of Alaska, Department of Public Works, Anchorage, Alaska.

Guymon, G.L., and Hromadka, T.V., 1982, Two-dimensional numerical model of coupled heat and moisture transport in frost heaving soils, final report: Contract DAAG29-19-C-0080, Army Research Office, Durham, N.C.

Hromadka, T.V. II, Guymon, G.L., and Pardoen, G.C., 1981, Nodal domain integration model of unsaturated two-dimensional soil water flow: development: Water Resources Research, v. 17, p. 1425-1430.

Hromadka, T.V. II, and Guymon, G.L., 1982, Application of a boundary integral equation to prediction of freezing fronts in soil: Cold Regions Sciences and Technology, v. 6, p. 115-121.

Ingersoll, J.I., Collins, C.M., Berg, R.L., and Gardner, C.R., 1979, 1978 field and laboratory studies on the Deadhorse, Alaska airfield: U.S. Army Cold Regions Research and Engineering Laboratory, Technical Note.

McKim, H.L., Anderson, D.M., Berg, R.L., and Tuinstra, R., 1975, Near real time hydraulic data acquisition utilizing the LANDSAT system, in Proceedings of the Conference on Soil-Water Problems in Cold Regions, Calgary, Alberta, p. 200-211.