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FIELD TESTS OF A FROST-HEAVE MODEL

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A one-dimensional mathematical model of frost heave based upon a nodal domain integration analog is compared to data collected from a Winchendon, Mass., field site. Air and soil temperatures, pore water pressures, and ground-water level data were collected on test sections containing six different soils during the winters of 1978-1979 and 1979-1980. The soil samples were evaluated in the laboratory to determine soil moisture characteristics, hydraulic conductivity as a function of pore water tensions, density, and other parameters. The parameters were used together with assumed thermal parameters in a one-dimensional model that calculates the distributions of temperature and moisture content as well as the amount of ice segregation (vertically lumped frost heave) and thaw consolidation. Using measured air and soil surface temperatures as input data, the simulated frost heave and thaw consolidation agreed well with measured ground surface displacements that resulted from ice segregation or ice lens melting.

This paper compares measured field data of frost heave and thaw consolidation and simulated frost heave and thaw consolidations using a one-dimensional model that is being developed by USACRREL for a project funded by the Corps of Engineers, the Federal Highway Administration, and the Federal Aviation Administration (Guymon et al. 1981a). This paper reports on continued verification of the mathematical model with some of the field data collected for the tri-agency project.

Beginning in the mid-1970's with Harlan (1973) and Guymon and Luthin (1974), investigators have increasingly studied the use of mathematical models to simulate coupled heat and moisture movement in freezing and thawing soils. In the early 1980's a number of investigators proposed using such models to estimate frost heave as well. Most of these models are reviewed by Guymon et al. (1980) and Hopke (1980). Generally, models are based upon deterministic equations that describe heat and moisture transport; however, the manner of incorporating required ancillary models, i.e. those used to determine latent heat effects, hydraulic properties, and stress distribution in the freezing zone, and the manner of assembling the complex models varies widely between investigators. It is reasonable to state that agreement on either the necessary level of complexity of frost heave models or the formulation of algorithms representing processes in the freezing zone is not wide-spread. Indeed, there is still considerable controversy associated with modeling endeavors. Due to the inherent spatial variability of soil properties, deterministic models will probably never be entirely satisfactory, and we have included probabilistic features in one version of the model used herein (Guymon et al. 1981b).

Probably the best way to show the utility of deterministic mathematical models of frost heave and the problems associated with their use is to compare results from these models with prototype field data and experimental laboratory data. We present such comparisons using a specific model that has been refined over several years.

THE NUMERICAL MODEL

The numerical model has been described elsewhere by Guymon et al. (1980, 1981a, 1981b, 1983) and Hromadka et al. (1982). Consequently, only a brief outline of the model is given here.

Table 1 presents the mathematical basis of the model. The nomenclature used in this table is defined at the end of this paper.

Phase-change processes in freezing or thawing zones are modeled by an isothermal approximation. The phase-change components for discrete times are decoupled using a simple control volume approach that accounts for the total available latent heat for freezing available water or thawing available ice. Overburden and surcharge effects are approximated by computing an equivalent positive pore water pressure, u, and adding it to simulated negative pore water pressures at discrete nodes where ice segregation is occurring. Positive pore water pressures due to consolidation are assumed to be negligible compared to overburden and surcharge stresses.

The equations of state are solved by a nodal domain integration analog (Hromadka et al. 1982), which results in a matrix system similar to the finite element system; that is, for the decoupled heat transport equation:

$$S\,T + C\,\dot{T} = F \qquad (1)$$

TABLE 1 Mathematical equation describing frost heave model.

Soil region	Energy state	Liquid moisture transport	Phase change	Sensible heat transport	Ancillary relationships
Surface boundary		$\partial(u-x)/\partial x = 0$		$T_u = T(t)$	
Frozen	$T < T_f$ $u = u(\theta_n) + \delta u_o$ $\delta = \begin{cases} 1, & \theta_i > \theta_o - \theta_n \\ 0, & \theta_i < \theta_o - \theta_n \end{cases}$	$\partial(u - x)/\partial x = 0 = \upsilon$	$\frac{\partial \theta_i}{\partial t} = 0$	$\frac{\partial}{\partial x}[K_T\, \partial T/\partial x] = C_m \frac{\partial T}{\partial t}$	$K_H = K(u) \cdot 10^{-E\theta_i},\ E\theta_i \geq 1$ $\theta_u = \theta(u)$ $\upsilon = -K_H \partial(u-x)/\partial x$ $C_m = [C_i\theta_i + C_w\theta_u + C_s(1-\theta_o)]/(1+\theta_s)$ $K_T = [K_i\theta_i + K_w\theta_u + K_s(1-\theta_o)]/(1+\theta_s)$ $\theta_s = \theta_i - (\theta_o - \theta_u),\ \theta_s > 0,\ \theta_u > \theta_n$ $y = \Sigma\theta_s \ell$
Freezing or thawing	$T = T_f$ $u(\theta_n) < u < 0$	$\frac{\partial \theta_u}{\partial t} = \frac{\partial}{\partial x}[K_H \partial(u-x)/\partial x]$	$-\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t}$ $L\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t}$	$+\frac{\partial}{\partial x}[K_T\, \partial T/\partial x] - C_w \upsilon \frac{\partial T}{\partial x} = C_m \frac{\partial T}{\partial t}$	
Unfrozen	$T > T_f$ $u(\theta_n) < u < u_L$	$\frac{\partial \theta_u}{\partial t} = \frac{\partial}{\partial x}[K_H \partial(u-x)/\partial x]$	$\frac{\partial \theta_i}{\partial t} = 0$	$\frac{\partial}{\partial x}[K_T\, \partial T/\partial x] - C_w \upsilon \frac{\partial T}{\partial x} = C_m \frac{\partial T}{\partial t}$	
Column bottom boundary		$u_L = u(t)$		$T_L = T(t) > T_f$	

where S is a matrix of conductivity parameters and spatial discretization, C is a matrix of capacitance parameters, T and $\dot{T}$ are the unknown state variable and temporal derivative vectors, and F is a vector of specified boundary conditions. The moisture transport equation reduces to an identical form.

The nodal domain integration method results in a C matrix that is also a function of a single mass lumping parameter, η; that is, C = C(η) where η can arbitrarily be chosen to represent various numerical analogs. For instance, if $\eta = 2$, a Galerkin finite element scheme results, or if $\eta \rightarrow \infty$, an integrated finite difference scheme results. The Galerkin finite element scheme was used in this study.

Governing equations are solved by decoupling them for discrete computational intervals and by holding each parameter constant. The solution is advanced in constant time-steps, Δt, by the Crank-Nicolson approximation. At the end of discrete time periods, nonlinear parameters are updated with ancillary relationships and secondary variables are computed. The computational technique and the sensitivity of results to the choice of numerical analog method are described in detail by Hromadka et al. (1982).

Guymon et al. (1981b, 1983) describe the verification of the model with freezing test data from laboratory soil columns, evaluate the uncertainty relating to boundary condition errors, describe modeling errors in general, and present a detailed analysis of parameter error effects.

FIELD AND LABORATORY DATA

Field data were collected at a test site in Winchendon, Massachusetts, located about 8 km (5 mi.) south of the New Hampshire border and about 32 km (20 mi.) east of the Connecticut River. The site consists of 26 asphalt concrete pavement test sections with 13 different test materials. The test sectons have a 76 mm (3 in.) asphalt concrete pavement underlain by a uniformly compacted subbase material from 0.9 to 1.5 m (3 to 5 ft) deep. Six of the test sections are included in the tri-agency study. Climatic data, ground-water levels, soil temperatures, and soil pore water pressure data were collected in the field during the 1978-1979 and 1979-1980 winters. Undisturbed and disturbed samples were obtained from the test soils and evaluated in the laboratory to determine the physical, hydraulic, and mechanical properties of the various materials. Field observations of frost heave, frost depth, and soil moisture tension were monitored for the following test materials: (a) Ikalanian silt, (b) Graves sandy silt, (c) Hart Brothers sand, (d) Sibley till, (e) Hyannis sand, and (f) dense-graded stone.

Figure 1 shows the mean daily air temperature beginning 10 December 1978 and extending through 15 March 1979. These data are determined from the average of the maximum and minimum daily temperatures taken from a thermograph. Several major freeze-thaw cycles occurred and, due to diurnal temperature variations, there were numerous diurnal freeze-thaw cycles during the winter. Table 2 presents the average monthly diurnal temperature variations.

Table 3 presents the physical data obtained in the laboratory. The soil water characteristics are represented by the equation

$$\theta = \frac{\theta_o}{A_w |u|^n + 1} \tag{2}$$

where the regression parameters A_w and n are given in Table 3. The freezing soil hydraulic conductivity factors, E, in Tables 1 and 3 were determined by calibration as described below. The E factor (Table 1) is a phenomenological parameter used to adjust unfrozen hydraulic conductivity to represent the hydraulic conductivity in the freezing zone. The version of the model used herein uses a hydraulic conductivity function in tabular form (i.e. hydraulic conductivity as a function of tension).

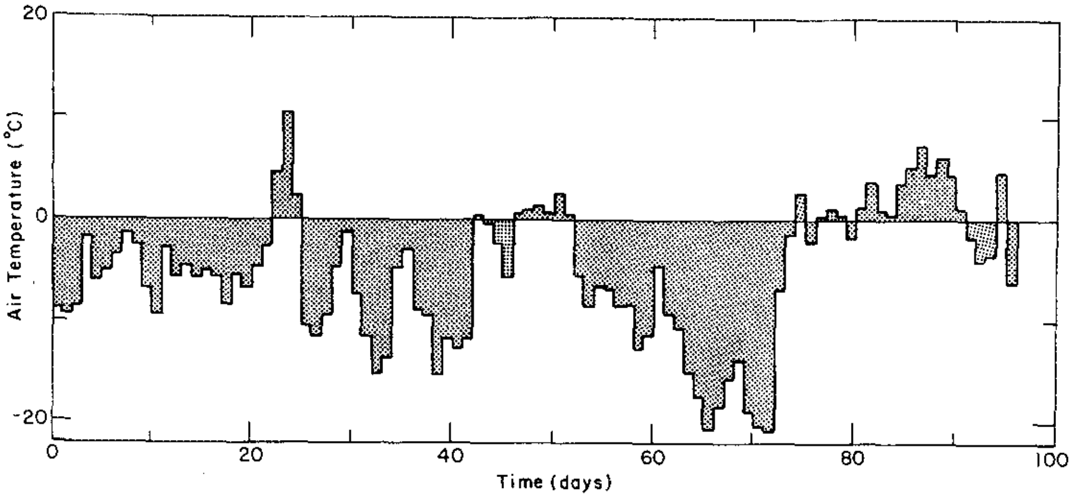

FIGURE 1 Mean daily air temperature, Winchendon, Mass., 10 Dec 1978 (day 0) through 15 Mar 1979.

TABLE 2 Average monthly diurnal temperature variations at the Winchendon test site, 1978-1979.

Temperature location	Mean daily amplitudes, °C			Max. daily amplitudes, °C			Min. daily amplitudes, °C			Mean daily amplitude coefficient of variation (%)		
	Jan	Feb	Mar	Jan	Feb	Mar	Jan	Feb	Mar	Jan	Feb	Mar
Air (1.5 m above g.s.)	4.7	7.4	7.3	10.3	14.4	18.3	0.6	0.7	1.4	51	50	58
Soil surface												
Ikalanian silt	4.6	9.2	7.4	8.5	11.4	16.6	0.6	6.6	2.2	50	20	58
Graves sandy silt	3.2	7.4	5.8	5.0	10.3	17.0	0.1	5.4	1.1	50	30	78
Hart Brothers sand	3.1	7.1	8.1	5.8	10.6	17.4	0.2	1.4	2.0	48	42	58
Sibley till	4.6	10.4	5.9	10.6	14.0	13.2	0.3	5.8	1.8	61	28	69

MODEL SIMULATION

A soil column 1 m long was assumed for all soils except Sibley till, where a 1.3 m column was used. Generally, the water table was a little over 1 m deep in all sections except the one containing Sibley till, which had a lower water table (less than 1.3 m). The soil column was divided into 50 variable- length elements ranging from 0.5 cm at the column top to 10 cm at the column bottom. A Crank-Nicolson time-step of 0.2 hr and a parameter update frequency of 1.0 hr were used. Column bottom boundary conditions were determined from recorded data. Mean daily surface temperature data or air temperature data multiplied by the Corps of Engineers n-factor (Berg 1974) were used for the upper boundary conditions. A sinusoidal diurnal surface temperature variation with a 7°C amplitude was used. Parameters were assumed, measured in the laboratory, or determined by calibration. Table 3 summarizes the parameters used in the simulations.

The results of simulations for the Graves sandy silt and Sibley till during the 1978-1979 winter are shown in Figures 2 and 3 along with the results when mean daily surface temperatures are used without a diurnal variation. Similar results were obtained for other soils studied. In general, errors introduced by using mean daily surface temperature were negligible. The most significant difference observed was for Graves sandy silt (Figure 2). In all cases, the use of mean daily surface temperatures approximated measured thaw penetration more accurately than the 7°C amplitude diurnal variation.

In some cases, it was difficult to predict accurately frost penetration near the end of the season. Measured frost (0°C isotherm) depths, which are subject to some error, are generally deeper than simulated with the model.

The only parameter calibrated was the freezing soil hydraulic conductivity correction factor. Adjusting other parameters, such as the soil water characteristics, might have yielded better overall

TABLE 3 Soil parameters for remolded Winchendon, Massachusetts, test site soils.

Parameter	Ikalanian silt	Graves sandy silt	Hart Bros. sand	Sibley till	Hyannis sand	Dense graded stone
Soil density (g/cm^3)[1]	1.70	1.49	1.690	1.970	1.690	1.870
Soil porosity (cm^3/cm^3)[1]	0.37	0.46	0.391	0.282	0.367	0.334
Soil-water freezing point depression (°C)[2]	0	0	0	0	0	0
Volumetric heat capacity of soil ($cal/cm^3/°C$)[2]	0.20	0.20	0.200	0.200	0.200	0.200
Thermal conductivity of soil (cal/cm/hr/°C)[2]	17.00	17.00	5.000	20.000	17.000	17.000
Unfrozen water content factor (cm^3/cm^3)[2]	0.03	0.12	0.040	0.150	0.010	0.150
Soil water characteristics [Aw,(n)] (unitless)[1]	0.000546 (1.5)	0.0056 (0.9)	0.022 (0.867)	0.062 (3.45)	0.00154 (1.806)	0.053 (0.462)
Saturated hydraulic conductivity (cm/hr)[1]	0.37	1.92	4.080	0.360	1.230	5.540
Freezing soil hydraulic conductivity factor (unitless)[3]	16.00	4.50	5.000	8.000	15.000	15.000

[1] Parameters measured in laboratory from remolded soil samples.
[2] Parameters were assumed.
[3] Parameters were calibrated.

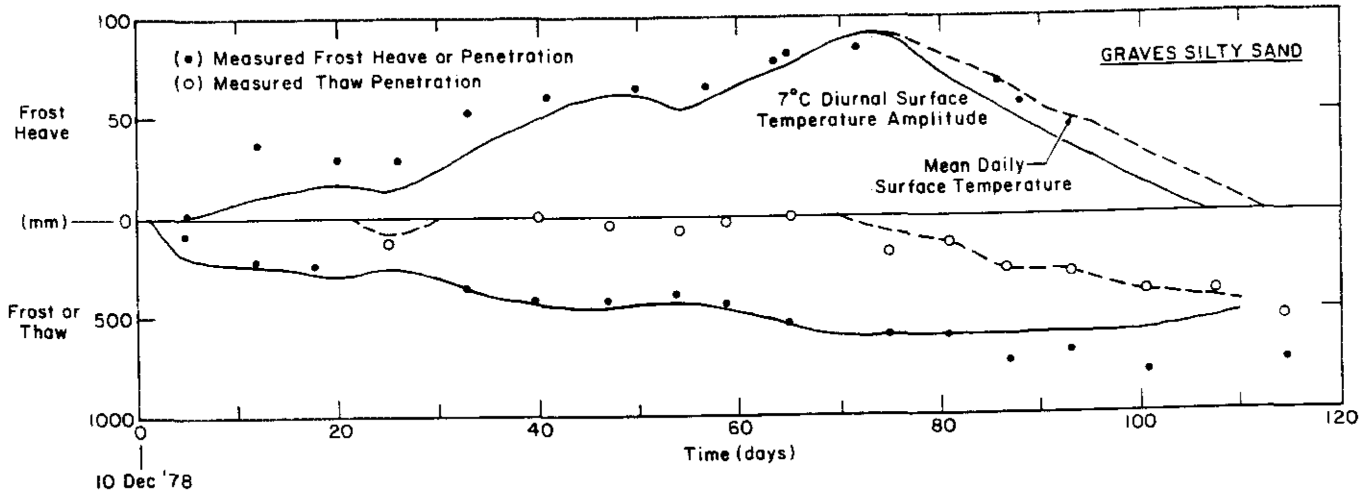

FIGURE 2 Simulated frost heave, thaw consolidation, frost penetration, and thaw penetration for Graves silty sand, 1978-1979 (day 0 = 10 Dec 1978).

results, but this type of calibration is not a wise procedure since it may mask errors in the model. Calibration errors may stem from three sources: (1) a surface moisture flux boundary condition error, (2) the fact that soil parameters vary due to freeze/thaw cycles, and (3) use of pavement surface temperatures instead of soil surface temperatures.

Alternate freezing and thawing occurs near the soil surface in most field prototype situations. It seems appropriate that any complete model of frost heave should include analogs that account for changes in key parameters, such as a hydraulic conductivity due to freeze/thaw cycles. Neither this model nor any other currently available attempts to

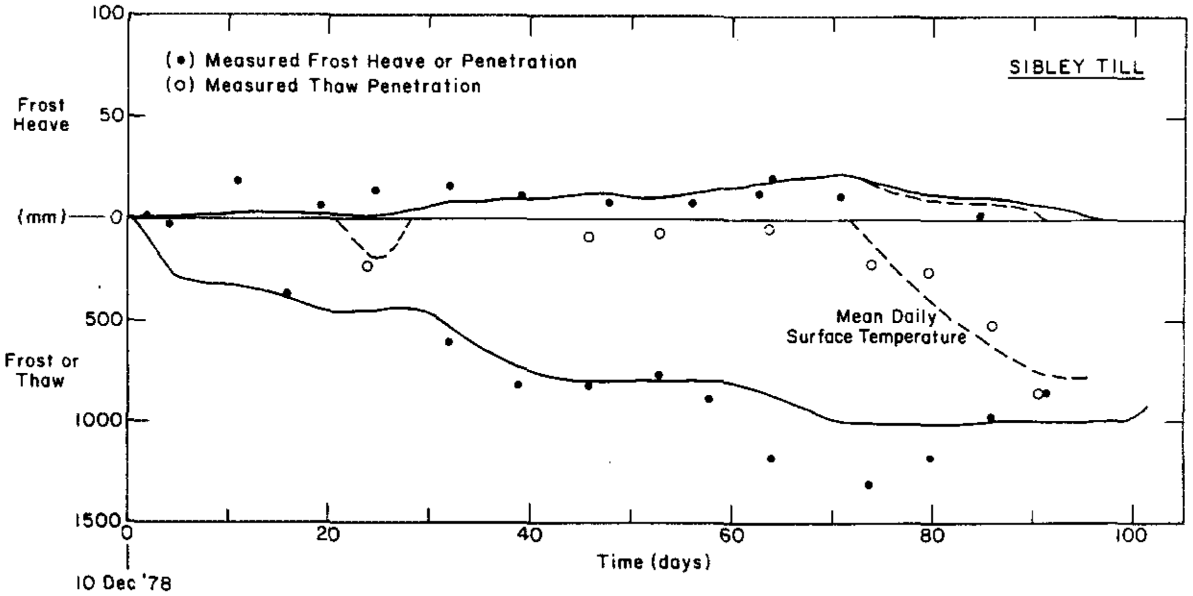

FIGURE 3 Simulated frost heave, thaw consolidation, frost penetration, and thaw penetration for Sibley till, 1978-1979 (day 0 = 10 Dec 1978).

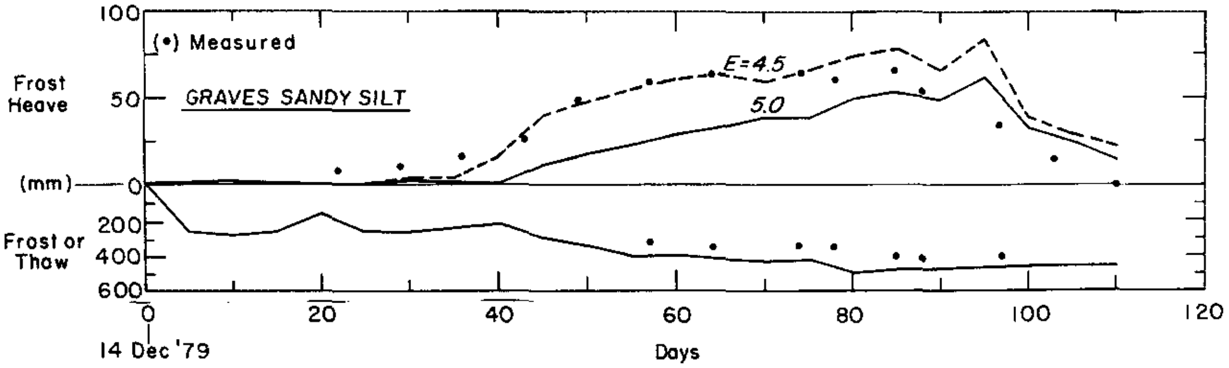

FIGURE 4 Simulated frost heave, thaw consolidation, frost penetration, and thaw penetration for Graves silty sand, 1979-1980.

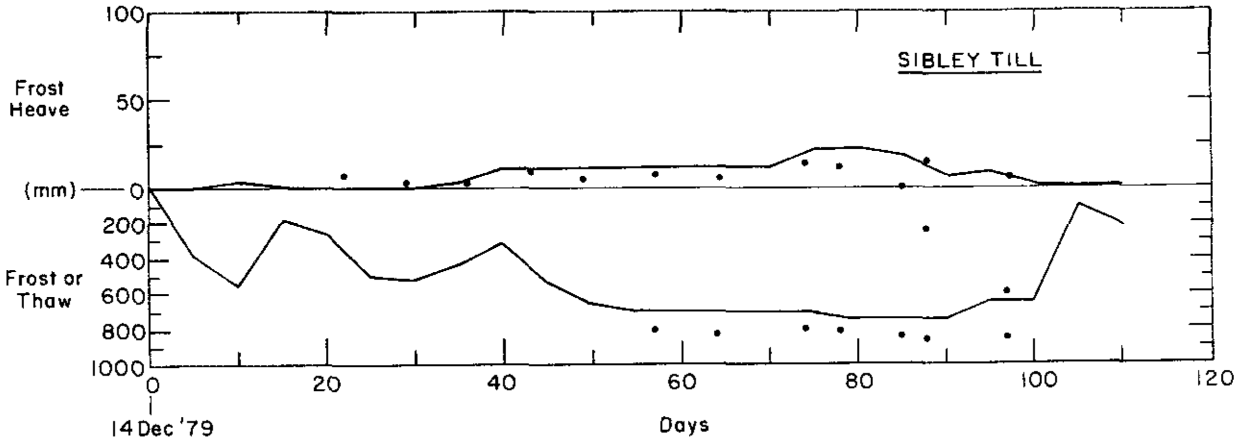

FIGURE 5 Simulated frost heave, thaw consolidation, frost penetration, and thaw penetration for Sibley till, 1979-1980.

include the influence of alternate freezing and thawing.

To further test the model, and particularly the validity of calibrating the E factor, dta for the 1979-1980 winter were used. Air temperature data and frost heave and frost penetration data are only available for (a) Ikalanian silt, (b) Graves sandy silt, (c) Hart Brothers sand, and (d) Sibley till.

Simulated results are compared to measured frost heave and frost penetration for Graves sandy silt and Sibley till in Figures 4 and 5. To match the measured maximum frost heave for the 1979-1980 winter more closely, the E factors are 10.3, 5.0, 9.0, and 8.5 for Ikalanian silt, Graves sandy silt, Hart brothers sand, and Sibley till, respectively. Data in Figure 4 show computed results using both E factors.

CONCLUSIONS

The proposed model can reasonably estimate field frost heave, provided accurate data on boundary conditions and parameters are available. Tests of the model were for field conditions where a number of freeze/thaw cycles occurred as well as significant diurnal temperature variations. Using measured air and soil surface temperatures as input data, the simulated frost heave and thaw consolidation agreed well with ground surface displacements resulting from ice segregation or ice lens melting.

Currently the model requires calibration to determine the E factor for each soil, the phenomenological parameter that accounts for decreased hydraulic conductivity in freezing or thawing zones. The results of the study reported here suggest that it is feasible to calibrate the model for field soils using imperfect input data; however, a long period of data is required with a number of freeze/thaw cycles.

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NOMENCLATURE

T = temperature (primary state variable)
u = pore water pressure (primary state variable)
θ_u = volumetric unfrozen water content (secondary state variable)
θ_i = volumetric ice content (secondary state variable)
θ_s = volumetric segregated ice content (secondary state variable)
x = coordinate vertically downward
ℓ = length of a finite element
y = lumped total frost heave (secondary state variable)
t = time
θ_n = unfrozen water content factor (parameter)
θ_o = soil porosity (parameter)
K_s = soil thermal conductivity (parameter)
C_s = soil volumetric heat capacity (parameter)
T_f = freezing point depression of water (parameter)
K_H = K_H (u) = hydraulic conductivity function (parameter)
E = freezing soil hydraulic conductivity correction (parameter)
K_i = ice thermal conductivity (specified constant)
K_w = water thermal conductivity (specified constant)
K_t = thermal conductivity of soil, water, ice mixture
C_i = ice volumetric heat capacity (specified constant)
C_w = water volumetric heat capacity (specified constant)
C_m = volumetric heat capacity of soil, water, ice mixture
L = bulk water volumetric latent heat coefficient (specified constant)
ρ_i = ice density (specified constant)
ρ_w = water density (specified constant)

Parameter refers to a measured or assumed parameter required in the model. Specific constant refers to a parameter obtained from standard tables. Variables are computed by the model.

REFERENCES

Berg, R.L., 1974, Energy balance on paved surface, Technical Report 226: Hanover, N.H., U.S. Army Cold Regions Research and Engineering Laboratory.

Guymon, G.L., and Luthin, J.N., 1974, A coupled heat and moisture transport model for arctic soils: Water Resources Research, v. 10, p. 995-1001.

Guymon, G.L., Hromadka, II, T.V., and Berg, R.L., 1980, A one-dimensional frost heave model based upon simulation of simultaneous heat and water flux: Cold Regions Science and Technology, v. 3, p. 253-263.

Guymon, G.L., Berg, R.L., Johnson, T.C., and Hromadka, II, T.V., 1981a, Results from a mathematical model of frost heave: Transportation Research Record, v. 809 p. 2-5.

Guymon, G.L., Harr, M.E., Berg, R.L., and Hromadka II, T.V., 1981b, A probabilistic-deterministic analysis of one-dimensioned ice segregation in freezing soil column: Cold Regions Science and Technology. v. 5, p. 127-140.

Guymon, G.L., Berg, R.L., Johnson, T.C., and Hromadka II, T.V., 1983, Mathematical model of frost heave in pavements: Hanover, N.H., U.S. Army Cold Regions Research and Engineering Laboratory, in preparation.

Harlan, R.L., 1973, Analysis of coupled heat-fluid transport in partially frozen soil: Water Resource Research, v. 9, p. 1314-1323.

Hopke, S.W., 1980, A model for frost heave including overburden: Cold Regions Science and Technology, v. 3, p. 111-147.

Hromadka II, T.V., Guymon, G.L., and Berg, R.L., 1982, Sensitivity of a frost heave model to the method of numerical simulation: Cold Regions Science and Technology, v. 6, p. 1-10.